Putting Research into Practice

From Our Curriculum Research Project

Classroom research showed that students might need help in understanding an expression as a conceptual object that has a particular structure. Expressions are the building blocks of algebra. Equations often are the major early focus of algebraic study, but an equation is just a statement that two expressions are equal. Understanding of algebraic language begins with learning to analyze and understand the structure of different expressions.

From Current Research: Equality

“What number would you put in the box to make this a true number sentence?” $8 + 4 = \square + 5$

Fewer than 10 percent of the students in any grade (1–6) gave the correct response of 7, and strikingly, performance did not improve with age. In fact, in this sample, results for the sixth-grade students were actually slightly worse than the results for the students in the earlier grades. These data illustrate that many, if not most, elementary school students harbor serious misconceptions about the meaning of the equal sign. Most elementary students, and many older students as well, do not understand that the equal sign denotes the relation between two equal quantities. Rather, they interpret the equal sign as a command to carry out a calculation, much as a calculator does when we press the equal sign. This misconception limits students’ ability to learn basic arithmetic ideas with understanding and their flexibility in representing and using those ideas, and it creates even more problems as they move to algebra.


Other Useful References: Algebra


Getting Ready to Teach Unit 7

Using the Common Core Standards for Mathematical Practice

The Common Core State Standards for Mathematical Content indicate what concepts, skills, and problem solving students should learn. The Common Core State Standards for Mathematical Practice indicate how students should demonstrate understanding. These Mathematical Practices are embedded directly into the Student and Teacher Editions for each unit in Math Expressions. As you use the teaching suggestions, you will automatically implement a teaching style that encourages students to demonstrate a thorough understanding of concepts, skills, and problems. In this program, Math Talk suggestions are a vehicle used to encourage discussion that supports all eight Mathematical Practices. See examples in Mathematical Practice 6.

**COMMON CORE Mathematical Practice 1**

*Make sense of problems and persevere in solving them.*

Students analyze and make conjectures about how to solve a problem. They plan, monitor, and check their solutions. They determine if their answers are reasonable and can justify their reasoning.

**TEACHER EDITION: Examples from Unit 7**

**MP.1 Make Sense of Problems**

*Check Answers* Encourage students to check the values they get when they evaluate their expressions to make sure they are reasonable in the context of the problem. If they are not, they will need to determine if their expression is incorrect or if they made an error in evaluating it.

**Lesson 3** Activity 2

**MP.1 Make Sense of Problems**

*Analyze Relationships* Give Student Pairs a few minutes to discuss and complete Exercise 2 on Student Book page 221. Then review the answers as a class. Be sure students recognize the relationship that consecutive terms of the pattern share. When described from left to right, this relationship is to add 5.

**Lesson 4** Activity 1

**Mathematical Practice 1** is integrated into Unit 7 in the following ways:

- Analyze Relationships
- Check Answers
- Draw a Diagram
COMMON CORE

Mathematical Practice 2
Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. They can connect diagrams and equations for a given situation. Quantitative reasoning entails attending to the meaning of quantities. In this unit, this involves translating expressions into words and reasoning about what location ordered pairs represent on the coordinate grid.

TEACHER EDITION: Examples from Unit 7

MP.2 Reason Abstractly and Quantitatively Connect Symbols and Words
Discuss the chart on Student Book page 215, which gives examples of expressions and how to say them in words. Read the text after the chart and Exercise 1, which both emphasize that there is more than one way to read an expression. Ask students to suggest other ways to read some of the other expressions in the chart. For example, 10 – 2 can be read as “Find 10 minus 2.”

MP.2 Reason Abstractly and Quantitatively
For Exercise 2, be sure students understand that the completed add 4 table represents two different vertical numerical patterns, and the terms in each row of the table are corresponding terms representing \( (x, y) \) ordered pairs.

Mathematical Practice 2 is integrated into Unit 7 in the following ways:
Connect Symbols and Words
COMMON CORE
Mathematical Practice 3
Construct viable arguments and critique the reasoning of others.

Students use stated assumptions, definitions, and previously established results in constructing arguments. They are able to analyze situations and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Students are also able to distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Students can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Talk is a conversation tool by which students formulate ideas and analyze responses and engage in discourse. See also Mathematical Practice 6: Attend to Precision.

TEACHER EDITION: Examples from Unit 7

MP.3 Construct Viable Arguments
Compare Models After the activity has been completed, give the class an opportunity to discuss the relationship that number lines share with finding horizontal and vertical distance in the coordinate plane. The axes and the grid lines of a coordinate plane are number lines. To find the distance between two points to the right of zero on any number line, subtract the lesser value from the greater value.

Lesson 5 ACTIVITY 1

What’s the Error? WHOLE CLASS
MP.3, MP.6 Construct Viable Arguments/Critique Reasoning of Others
Puzzled Penguin In this activity, Puzzled Penguin analyzes only the relationships shared by the first three terms of the pattern. By ignoring the remaining terms, Puzzled Penguin does not realize that the rule of adding 1, adding 2, and so on, is not true for all of the given terms. Upon completion of the activity, remind students that they must analyze all of the given terms of a numerical pattern before identifying the rule of the pattern.

Lesson 4 ACTIVITY 2

Mathematical Practice 3 is integrated into Unit 7 in the following ways:

Compare Models
Compare Representations
Puzzled Penguin
Mathematical Practice 4
Model with mathematics.

Students can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an equation to solve a problem. Students might draw diagrams to lead them to a solution for a problem. Students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation. They are able to identify important quantities in a practical situation and represent their relationships using such tools as diagrams, tables, graphs, and formulas.

**TEACHER EDITION: Examples from Unit 7**

**MP.4 Model with Mathematics** Write an Expression Complete Part a of Problem 13 as a class. Ask:
- How do we find the total amount the friends earned? Add $24 and $m$ dollars. What expression shows this total? $24 + m$
- How do we find how much each friend gets? Divide the total by 4. How do we write this as an expression? $(24 + m) \div 4$
- Why do we need the parentheses? To show that we add to find the total amount first and then divide.
- Part b says the friends made $50 mowing lawns. How can we find the amount each friend gets? Substitute 50 for $m$ and then simplify the expression.

Ask pairs to finish Part b and then choose a volunteer to present the solution.

**MP.1, MP.4 Make Sense of Problems/Model with Mathematics** Draw a Diagram In Problem 2, one point of the graph is located on the $x$-axis. Be sure students understand that any point on the $x$-axis has a $y$-coordinate of 0 and any point on the $y$-axis has an $x$-coordinate of 0.

Mathematical Practice 4 is integrated into Unit 7 in the following ways:
- Draw a Diagram
- Write an Expression
**Mathematical Practice 5**

**Use appropriate tools strategically.**

Students consider the available tools and models when solving mathematical problems. Students make sound decisions about when each of these tools might be helpful. These tools might include paper and pencil, a straightedge, a ruler, or the MathBoard. They recognize both the insight to be gained from using the tool and the tool’s limitations. When making mathematical models, they are able to identify quantities in a practical situation and represent relationships using modeling tools such as diagrams, grid paper, tables, graphs, and equations.

Modeling numbers in problems and in computations is a central focus in *Math Expressions* lessons. Students learn and develop models to solve numerical problems and to model problem situations. Students continually use both kinds of modeling throughout the program.

**TEACHER EDITION: Examples from Unit 7**

**MP.5 Use Appropriate Tools**

**Coordinate Plane Poster**

Use the Coordinate Plane Poster or make a transparency of Coordinate Grid (TRB M23), and display it on the overhead. Somewhere next to the grid write “(x, y) means (→,↑).”

Invite a number of volunteers to each write an ordered pair, such as (3, 4), on the board. Make sure that at least one point on each axis is included such as (0, 2) and (4, 0).

Then invite other volunteers to each choose a point listed on the board and plot and label that point on the poster or transparency. For each point, have the volunteer explain how he or she found its location.

**MP.5 Use Appropriate Tools**

**Straightedge**

Students will have an easier time visualizing the constellation if they use a straightedge to connect the points.

**Mathematical Practice 5** is integrated into Unit 7 in the following ways:

- Coordinate Plane Poster
- Make a Graph
- Straightedge
ACTIVITY 3

ACTIVITY 1

UNIT 7 MATH BACKGROUND

Mathematical Practice 6
Attend to precision.

Students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. They are careful about specifying units of measure to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. Students give carefully formulated explanations to each other.

MP.6 Attend to Precision Explain Solutions When students have finished the exercises on Student Book page 217, ask volunteers to present the solutions. Address any difficulties or misunderstandings students had.

MP.6 Attend to Precision The sentences students write to support their interpretation of the value of the expression \(5 + 3(4 \div 2) - 1\) must include precise mathematical words, and must include precise facts and symbols.

MATH TALK Invite students to make a conjecture about how \(x\)-coordinates can be used to find the length of side \(MJ\) or side \(LK\). To find horizontal distance in the coordinate plane, subtract the \(x\)-coordinates of the points. Ask students to test their conjecture by using it to find the length of the segment and comparing the result to the answer produced by counting. Subtracting the \(x\)-coordinates and counting produce the same result—the length of side \(MJ\) or side \(LK\) is 2 units.

MATH TALK Use this discussion to connect real world situations to numerical patterns.

What other scenarios in our lives involve the idea of numerical patterns?

Savannah: Sometimes when I buy something at a store, I have to pay sales tax. The tax is a number pattern because for every dollar I spend, I am charged a certain amount of tax.

Pete: And if I spend twice the amount of money Savannah does, I pay twice as much tax. It doesn’t matter what anyone spends, it’s still a pattern.

Juan: This morning I paid 15¢ for a pencil in the school bookstore. If I bought two pencils, it would have been 30¢, and 45¢ if I bought three. So that’s a real world numerical pattern too.

How could you graph the scenarios you described, and what might the graph look like?

Juan: Since pencils are 15¢ each, I would plot a point at \((0, 0)\) because there’s no cost if you don’t buy any pencils, and then I’d plot points at \((1, 15)\), \((2, 30)\), and so on. You would see the graph as rising from left to right.

Mathematical Practice 6 is integrated into Unit 7 in the following ways:

Explain Solutions | Puzzled Penguin
**Mathematical Practice 7**

*Look for and make use of structure.*

Students analyze problems to discern a pattern or structure. They draw conclusions about the structure of the relationships they have identified.

**TEACHER EDITION: Examples from Unit 7**

**MP.7 Look for Structure** Refer students to Student Book page 218. Read the text about brackets and braces at the top. Copy the expression $12 \cdot [48 \div (4 + 4)]$ from the example onto the board. Point out that the expression in parentheses is *nested* inside the expression in brackets. Explain that, in expressions like this, we work from the inside out. Simplify the expression on the board, asking students what to do at each step.

**Mathematical Practice 7** is integrated into Unit 7 in the following ways:

**Identify Relationships**
Mathematical Practice 8

Look for and express regularity in repeated reasoning.

Students use repeated reasoning as they analyze patterns, relationships, and calculations to generalize methods, rules, and shortcuts. As they work to solve a problem, students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**MP.8 Use Repeated Reasoning**

**Generalize** Emphasize that evaluating an expression involves two steps.

- Replace the variable with the given number.
- Use the Order of Operations to simplify the expression you get in the first step.

Students should notice that once they have substituted the number for the variable, they have a numerical expression.

**Mathematical Practice 8** is integrated into Unit 7 in the following ways:

**Generalize**

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**STUDENT EDITION: LESSON 7, PAGES 231–232**

**Focus on Mathematical Practices**

Unit 7 includes a special lesson that involves solving real world problems and incorporates all 8 Mathematical Practices. In this lesson, students use what they know about ordered pairs to graph constellations.
Getting Ready to Teach Unit 7

Learning Path in the Common Core Standards
In this unit, students use algebraic reasoning and coordinate graphs. Their work involves reading, writing, simplifying, and evaluating algebraic expressions, exploring patterns and relationships, and plotting and locating points in the coordinate plane.

Visual models and real world situations are used throughout the unit to illustrate algebraic thinking.

Help Students Avoid Common Errors
Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- **Lesson 2:** Not writing parentheses to indicate the operation to be performed first
- **Lesson 4:** Incorrectly predicting a term in a pattern
- **Lesson 5:** Transposing x- and y-coordinates in the coordinate plane

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.
Students begin to build their ability to reason algebraically by working with expressions. Expressions use numbers and symbols (such as variables and operation signs) to “express” computations.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Computation in Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 + 6.3</td>
<td>Add 3.5 and 6.3.</td>
</tr>
<tr>
<td>10 – 2</td>
<td>Subtract 2 from 10.</td>
</tr>
<tr>
<td>( \frac{1}{2} \cdot p )</td>
<td>Multiply ( p ) by ( \frac{1}{2} ).</td>
</tr>
<tr>
<td>14 ÷ 4</td>
<td>Divide 14 by 4.</td>
</tr>
</tbody>
</table>

Students write a computation in words given an expression, and write an expression given a computation in words. Because addition and multiplication are commutative operations, symbolic and word expressions for addition and multiplication can be written in different ways.

Add 14 and \( t \). \( \rightarrow 14 + t \) or \( t + 14 \)

Find the product of 12 and 0.1. \( \rightarrow 12 \times 0.1 \) or \( 0.1 \times 12 \)

Parentheses In Lessons 8–11 in Unit 6, students translated two-step and multistep word problems to equations, and learned the importance of using grouping symbols to indicate the operation to be performed first if an equation contained two or more different operations.

The concept of using parentheses as grouping symbols is emphasized in the second activity in Lesson 1, and in Lessons 2 and 3 of this unit.

Express \( 8 + 27 \) + 2 or \( 6 \times 30 \) + \( 6 \times 7 \). Note however that the numbers in expressions need not always be whole numbers.
Expressions with More than One Operation  Lesson 1 also introduces students to expressions that involve more than one operation and the concept of the Order of Operations.

**Order of Operations**

*Step 1* Perform operations inside parentheses.
*Step 2* Multiply and divide from left to right.
*Step 3* Add and subtract from left to right.

Students use the Order of Operations to indicate, by using words or by writing an expression with parentheses, the operation to be performed first.

10. Consider the expression $12 \div (5 + 2)$.
   a. Which operation is done first, division or addition?
      - addition
   b. Write the computation in words.
      - Possible answer: Divide 12 by the sum of 5 and 2.

11. Consider the expression $12 \div 5 + 2$.
   a. Which operation is done first, division or addition?
      - division
   b. Write the computation in words.
      - Possible answer: Divide 12 by 5 and then add 2.

Write the computation in words or the expression for the words. Think about the Order of Operations.

12. $3.5 - (2.1 + 1.2)$ Subtract the sum of 2.1 and 1.2 from 3.5.
13. $\frac{1}{2} + \frac{3}{4} \cdot t$ Add $\frac{1}{2}$ to the product of $\frac{3}{4}$ and $t$.
14. $(25 - 10) \div 5$ Subtract 10 from 25 and then divide by 5.
15. Multiply the sum of $p$ and 3 by 0.1. $(p + 3) \cdot 0.1$ or $0.1 \cdot (p + 3)$

A great deal of the mathematics students will learn and perform in their future studies will involve, in some way, the Order of Operations.
**Simplify Expressions** The activities in Lesson 2 involve simplifying expressions, and students learn that simplifying an expression means finding its value.

A common error when simplifying expressions is to ignore the Order of Operations and instead, perform the operations as they appear from left to right. It is important for students to understand many expressions will not simplify correctly if the Order of Operations is ignored. For example, the expression below simplifies to 18 if the operations are performed left to right.

$$12 - 3 \cdot 2$$

To accustom themselves to following the Order of Operations, students first simplify an expression in a step-by-step fashion.

1. Follow the Order of Operations to simplify $25 - (5 + 2) \cdot 3$.
   
   **Step 1** Perform operations inside parentheses. $25 - 7 \cdot 3$
   
   **Step 2** Multiply and divide from left to right. $25 - 21$
   
   **Step 3** Add and subtract from left to right. $4$

Students then simplify expressions without prompts, gaining experience working within the Order of Operations.

**Simplify. Follow the Order of Operations.**

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<thead>
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</thead>
<tbody>
<tr>
<td>2. $5 + 16 ÷ 4$</td>
<td>3. $10 \cdot (0.3 + 0.2)$</td>
<td>4. $20 ÷ 4 + 3 \cdot 3$</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>(5/6 - 1/3) \cdot 4</td>
<td>6. $21 - 12 + 9 - 2$</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>8. $0.3 + 0.1 \cdot 5 + 0.2$</td>
<td>9. $18 + 9 ÷ 0.1$</td>
<td>10. $36 ÷ 3 \cdot 2$</td>
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<tr>
<td>1</td>
<td>108</td>
<td>24</td>
</tr>
</tbody>
</table>

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**from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

**Connection to Expressions and Equations** In Grade 6, students will begin to view expressions not just as calculation recipes but as entities in their own right, which can be described in terms of their parts. For example, students see $8 \cdot (5 + 2)$ as the product of 8 with the sum $5 + 2$. In particular, students must use the conventions for order of operations to interpret expressions, not just to evaluate them. Viewing expressions as entities created from component parts is essential for seeing the structure of expressions in later grades and using structure to reason about expressions and functions.

As noted above, the foundation for these later competencies is laid in Grade 5 when students write expressions to record a "calculation recipe" without actually evaluating the expression, use parentheses to formulate expressions, and examine patterns and relationships numerically and visually on a coordinate plane graph. Before Grade 5, student thinking that also builds toward the Grade 6 [Expressions & Equations] work is focusing on the expressions on each side of an equation, relating each expression to the situation, and discussing the situational and mathematical vocabulary involved to deepen the understandings of expressions and equations.
Evaluate Expressions  In Lesson 3, students’ algebraic reasoning skills evolve to include expressions with variables. Variables represent unknown numbers. The concept of variables is not new to students. They have worked with formulas that involve variables in the past, such as using $A = l \cdot w$ and $P = 2l + 2w$ to compute the area and perimeter of a rectangle.

In this lesson students learn about evaluating expressions. Evaluating an expression means substituting values for the variables and then performing the operations using the Order of Operations.

Evaluate $5 \cdot (p - 2)$ for $p = 10$.

$$5 \cdot (p - 2) = 5 \cdot (10 - 2)$$ Substitute 10 for $p$.
$$= 5 \cdot 8$$ Subtract inside parentheses.
$$= 40$$ Multiply.

The expressions students evaluate include fractions, decimals and whole numbers.

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<thead>
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<tbody>
<tr>
<td>1.</td>
<td>$m - 4.7$ for $m = 10$</td>
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<td></td>
<td>5.3</td>
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<tr>
<td>2.</td>
<td>$5 \div x$ for $x = \frac{1}{3}$</td>
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<td></td>
<td>15</td>
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<tr>
<td>3.</td>
<td>$5 + n \cdot 4$ for $n = 3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>17</td>
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<tr>
<td>4.</td>
<td>$\frac{1}{5} \cdot x$ for $x = 15$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>$7.5 \times (d - 2.5)$ for $d = 3.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>$48 \div (z - 6)$ for $z = 14$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>$10 \cdot (0.05 + q)$ for $q = 1.2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.5</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>$\frac{3}{4} + d - 1\frac{1}{4} + 5\frac{1}{2}$ for $d = 1\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$8\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>$1,000 \cdot h$ for $h = 0.004$</td>
<td></td>
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<tr>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>$(t + 18) \div 5$ for $t = 17$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>$54 \div 3 \cdot v$ for $v = 3$</td>
<td></td>
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<tr>
<td></td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>$6 \cdot 0.01 + n \cdot 0.1$ for $n = 2$</td>
<td></td>
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<tr>
<td></td>
<td>0.26</td>
<td></td>
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</tbody>
</table>
Writing Expressions  A variety of real world contexts provide students with opportunities to apply their understanding of expressions.

13. Four friends earned $24 by washing cars and $m$ dollars by mowing lawns. They want to divide the total equally.
   a. Write an expression for the amount each friend gets.
      \((24 + m) \div 4\)
   b. If they made $50 mowing lawns, how much should each friend get?
      $18.50

14. There are \(\frac{2}{3}\) as many students in science club as in math club.
   a. If there are \(m\) students in math club, how many are in science club?
      \(\frac{2}{3} \cdot m\)
   b. If there are 27 students in math club, how many are in science club?
      18

15. Kima’s cat weighs 6 pounds more than her rabbit. Her dog weighs 3 times as much as her cat. Let \(r\) be the weight of Kima’s rabbit.
   a. How much does her cat weigh?
      \(r + 6\) pounds
   b. How much does her dog weigh?
      \(3 \cdot (r + 6)\) pounds
   c. If Kima’s rabbit weighs 5 pounds, how much do her cat and dog weigh?
      cat: 11 pounds; dog: 33 pounds

16. To change a temperature from degrees Celsius to degrees Fahrenheit, multiply it by \(\frac{9}{5}\) and then add 32.
   a. Let \(c\) be a temperature in degrees Celsius. Write an expression for changing \(c\) to degrees Fahrenheit.
      \(\frac{9}{5} \cdot c + 32\)
   b. Use your expression to change 20°C to Fahrenheit degrees.
      68°F
**Patterns and Relationships**

**Patterns** Work with patterns begins with numerical sequences (or progressions) of numbers. In each sequence, consecutive numbers (i.e., terms) share the same relationship.

The initial sequence 3, 5, 7, 9, 11, . . . is an example of an arithmetic sequence. A sequence is arithmetic if the difference between consecutive terms is constant.

\[
\begin{array}{cccccc}
3 & 5 & 7 & 9 & 11 \\
3 & 3 + 2 & 3 + 2 + 2 & 3 + 2 + 2 + 2 & 3 + 2 + 2 + 2 + 2 \\
3 & 3 + (1 \cdot 2) & 3 + (2 \cdot 2) & 3 + (3 \cdot 2) & 3 + (4 \cdot 2)
\end{array}
\]

Numerical expressions can be written to describe the terms of the sequence above. Writing expressions gives students a way to find the sixth term of this sequence, or the next term of any sequence. The expressions used to predict 13, the next term of the sequence above, are \(3 + 2 + 2 + 2 + 2 + 2\) or \(3 + 5 \cdot 2\).

Students begin their work with numerical sequences by being given the rule of the sequence.

2. **a.** Write the first five terms of a numerical pattern that begins with 5 and then adds 5.
   \[5, 10, 15, 20, 25\]

   **b.** Write an expression for the sixth term of the pattern.
   \[5 + 5 + 5 + 5 + 5 + 5 \text{ or } 6 \cdot 5\]

   **c.** Write the sixth term. \[30\]

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*from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING*

**Patterns** Students extend their Grade 4 pattern work by working briefly with two numerical patterns that can be related and examining these relationships within sequences of ordered pairs and in the graphs in the first quadrant of the coordinate plane. This work prepares students for studying proportional relationships and functions in middle school.
Patterns and Relationships  Students progress from working with one number sequence to working with two. Working with two number sequences gives students an opportunity to think in a proportional way. This work helps form the conceptual foundation for their future studies involving functions and a variety of proportional relationships.

An important part of working with two number sequences is identifying the corresponding terms of the sequences. To identify the corresponding terms, students need to view the terms of the patterns in an ordinal sense, in other words, as positional. All terms occupying the same position are corresponding terms, so the terms in the first position are corresponding, the terms in the second position are corresponding, and so on. Drawing circles helps students see the corresponding terms and understand how they are related.

4. a. Write the first five terms of a pattern that begins with 2, and then adds 2.
   
   b. Write the first five terms of a pattern that begins with 4, and then adds 4.
   
   c. Circle the corresponding pairs of terms in the patterns. How does each term in the top pattern compare to the corresponding term in the bottom pattern?
   
   The top term is the bottom term divided by 2.

   d. How does the bottom term compare to the top term?
   
   The bottom term is the top term times 2.

6. a. Write the first five terms of two different patterns.  Sample answers shown.
   
   b. Circle the corresponding terms in your patterns. Then describe two different relationships that the corresponding terms of your patterns share.
   
   The top term is the bottom term divided by 10; the bottom term is the top term times 10.
Real World Patterns  Students also work with real world examples of number patterns. Presenting the data in tables makes it easier for students to locate the corresponding terms and identify the relationship those terms share.

<table>
<thead>
<tr>
<th>Overdue Book Late Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Days Late</td>
</tr>
<tr>
<td>Late Fee</td>
</tr>
</tbody>
</table>

7. Describe the relationship between the corresponding terms.

Sample answer: The late fee in cents is the number of days late multiplied by fifteen.

Complete the table and describe the relationship between corresponding terms.

<table>
<thead>
<tr>
<th>Bicycles and Wheels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycles</td>
</tr>
<tr>
<td>Wheels</td>
</tr>
</tbody>
</table>

Sample answer: The number of wheels is the number of bicycles multiplied by two.

<table>
<thead>
<tr>
<th>Cost of Concert Tickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tickets</td>
</tr>
<tr>
<td>Cost in Dollars</td>
</tr>
</tbody>
</table>

Sample answer: The cost of tickets in dollars is the number of tickets multiplied by thirty-five.

<table>
<thead>
<tr>
<th>Weather Relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inches of Rain</td>
</tr>
<tr>
<td>Inches of Snow</td>
</tr>
</tbody>
</table>

Sample answer: The number of inches of snow is the number of inches of rain multiplied by ten.
The Coordinate Plane  In Lesson 5 students are introduced to the coordinate plane. The plotting and locating of points in the plane is an extension of their work with the proportional relationships of number sequences in Lesson 4.

A coordinate plane is a two-dimensional system that uses numerical coordinates to specify points, or locations, in the plane. It is important for students to understand the coordinates of the form \((x, y)\) as measures of distance, with the \(x\)-coordinate representing perpendicular distance from the \(y\)- or vertical axis, and the \(y\)-coordinate representing perpendicular distance from the \(x\)- or horizontal axis. In Grade 5, work in the coordinate plane is limited to Quadrant I.

In addition to plotting and locating points in the plane, Problem 31 makes students aware of the common error of transposing the coordinates of an ordered pair as they identify and explain to Puzzled Penguin how to correct the error.

31. Write a response to Puzzled Penguin.

No, point \((7, 4)\) is not correct because it would not form a square. You listed the coordinates in the wrong order. The correct point has an \(x\)-coordinate of 4 and a \(y\)-coordinate of 7. Its ordered pair would be \((4, 7)\).
The coordinate plane activities also include those that are more open-ended.

Plot and label a point at each location.

20. point P at (7, 10)  21. point C at (0, 6)  22. point Z at (9, 7)

On the coordinate plane above, draw an angle of the given type. The angle should have its vertex at one labeled point and sides that pass through two other labeled points. Give the name of the angle. Answers will vary; sample answers shown.

23. acute angle \( \angle QVC \)  24. obtuse angle \( \angle PZR \)  25. right angle \( \angle CWJ \)

Students also compute horizontal and vertical distance.

26. Explain how subtraction can be used to find the lengths of line segments \( AB \) and \( AC \).

To find the length of segment \( AB \), subtract the \( y \)-coordinates of \( B \) from \( A \); \( 10 - 7 = 3 \). To find the length of segment \( AC \), subtract the \( x \)-coordinates of \( A \) from \( C \); \( 8 - 1 = 7 \).
Generate and Graph Ordered Pairs  The activities in Lesson 6 are again related to the concept of proportional relationships. Although students are not told they are working with functions in this lesson, they are working with a type of function known as a linear function. The graph of a linear function is a straight line.

Numerical patterns can be written horizontally or vertically. The add 4 table below shows a numerical pattern in the left column and the result of adding 4 in the right column.

<table>
<thead>
<tr>
<th>add 4</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 5</td>
<td>(1, 5)</td>
</tr>
<tr>
<td>2 6</td>
<td>(2, 6)</td>
</tr>
<tr>
<td>3 7</td>
<td>(3, 7)</td>
</tr>
<tr>
<td>4 8</td>
<td>(4, 8)</td>
</tr>
<tr>
<td>5 9</td>
<td>(5, 9)</td>
</tr>
</tbody>
</table>

1. Complete the add 4 table.
2. Complete the (x, y) table to show the ordered pairs that the add 4 table represents.
3. Each ordered pair represents a point in the coordinate plane. Graph and connect the points.
**Real World Connections**  A constant change over time is a real world example of a proportional relationship. Students complete their work with proportional relationships by solving real world problems.

The graph represents an automobile traveling at a constant speed.

9. The points on the graph represent four ordered \((x, y)\) pairs. Write the ordered pairs.

\[(0, 0), (1, 60), (2, 120), (3, 180)\]

10. Complete the table to show the relationship between time and distance.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (miles)</td>
<td>0</td>
<td>60</td>
<td>120</td>
<td>180</td>
</tr>
</tbody>
</table>

11. At what constant rate of speed was the automobile traveling? Explain how you know.

60 mph; Sample explanation: The graph passes through the point at (1 hour, 60 miles).

**Focus on Mathematical Practices**

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use coordinate graphs to explore constellations.