Adding It Up: Helping Children Learn Mathematics, a report published by the National Research Council in 2001, is a summary of the work of a committee of members from diverse backgrounds who reviewed and synthesized relevant research on mathematics learning from pre-kindergarten through grade 8 (National Research Council [NRC], 2001, p. ix).

The committee chose the term mathematical proficiency to capture what it means for anyone to learn mathematics successfully. Mathematical proficiency is broken down into the following five components, or strands:

- **conceptual understanding**—comprehension of mathematical concepts, operations, and relations
- **procedural fluency**—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **strategic competence**—ability to formulate, represent, and solve mathematical problems
- **adaptive reasoning**—capacity for logical thought, reflection, explanation, and justification
- **productive disposition**—habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one’s own efficacy.

The most important observation the committee makes and stresses is that the five strands are interwoven and interdependent in the development of proficiency in mathematics (NRC, 2001, p. 116).
*Houghton Mifflin Math* is based on the premise that the five strands of mathematical proficiency are interwoven. Pages 314 and 315 from grade 5 of this program can be used to illustrate this.

- The concept of multiplication of fractions is developed by connecting the fraction-bar model with the area model used for multiplication of whole numbers (*conceptual understanding*).

- The concrete model is used to develop the algorithm which is then practiced in the exercises (*procedural fluency*).

- Word problems involving multiplication of fractions are included in the lesson (*strategic competence*).

- Two problems subtitled Write About It and Explain require students to think logically, reflect, explain, and justify (*adaptive reasoning*).

- The word problems center around a school play, a topic that is age-appropriate (*productive disposition*).

This framework of mathematical proficiency, based on the five strands, is also similar to the framework used in recent mathematics assessments by the National Assessment of Educational Progress (NAEP), which features three mathematical abilities (conceptual understanding, procedural knowledge, and problem solving) and includes additional specifications for reasoning, connections, and communication (National Assessment Governing Board, 2000).