Math Talk

A significant part of the collaborative classroom culture is the frequent exchange of mathematical ideas and problem-solving strategies, or Math Talk. There are multiple benefits of Math Talk:

- Describing one’s methods to another person can clarify one’s own thinking as well as clarify the matter for others.
- Another person’s approach can supply a new perspective, and frequent exposure to different approaches tends to engender flexible thinking.
- In the collaborative Math Talk classroom, students can ask for and receive help, and errors can be identified, discussed, and corrected.
- Student math drawings accompany early explanations in all domains, so that all students can understand and participate in the discussion.
- Math Talk permits teachers to assess students’ understanding on an ongoing basis. It encourages students to develop their language skills, both in math and in everyday English.
- Math Talk enables students to become active helpers and questioners, creating student-to-student talk that stimulates engagement and community.

To encourage Math Talk, teachers can stand at the side or back of the classroom to help students interact more directly with each other. Teachers say that it is necessary for them to “bite their tongue” to keep from doing all of the talking. Student voices and explanations will emerge if you wait. For new topics, teachers may need to model explaining so that students learn to use new vocabulary; however, some students can usually explain even for a new topic.

The key supports for Math Talk are the various participant structures, or ways of organizing class members as they interact. The teacher always guides the activity to help students work both as a community and also independently. Descriptions of the most common participant structures follow.

Math Talk Participant Structures

Solve and Discuss (Solve, Explain, Question, and Justify) at the Board

The teacher selects 4 to 5 students (or as many as space allows) to go to the classroom board and solve a problem, using any method they choose. Their classmates work on the same problem at their desks. Then the teacher picks 2 or 3 students to explain their methods. Students at their desks are encouraged to ask questions and to assist their classmates in understanding.

Benefits: Board work reveals multiple methods of solving a problem, making comparisons possible, and communicating to students that different methods are acceptable. The teacher can select methods to highlight in subsequent discussions. Spontaneous helping occurs frequently by students working next to each other at the board. Time is used efficiently because everyone in the class is working. In addition, errors can be identified in a supportive way and corrected and understood by students.

Small Group Version of Solve and Discuss

Students can solve individually in small groups and then 2 or 3 students explain their method while the group members ask questions and help the explanation to be very clear. On the next problem different group members explain.

Benefits: Everyone gets a chance to explain and be helped in their explanation. This approach may work better for a given topic after whole-class discussions of the topic have taken place.
Student Pairs
Two students work together to solve a problem, to explain a solution method to each other, to role play within a mathematical situation (for example, buying and selling), to play a math game, or to help a partner. They are called Helping Pairs when more advanced students are matched with students who are struggling. Pairs may be organized formally, or they may occur spontaneously as help is needed. Initially, it is useful to model pair activities, contrasting effective and ineffective helping.

Benefits: Pair work supports students in learning from each other, particularly in applying and practicing concepts introduced in whole-class discussion. Helping Pairs often foster learning by both students as the helper strives to adopt the perspective of the novice. Helping almost always enables the helper to understand more deeply.

Scenarios
The main purpose of scenarios in grades K–2 is to demonstrate mathematical relationships in a visual and memorable way. In scenario-based activities a group of students is called to the front of the classroom to act out a particular mathematical situation.

Benefits: The scenario structure often fosters a sense of involvement among students. Scenarios also create meaningful contexts in which children can reason about numbers and relate math to their everyday lives.

Small Groups
Unstructured groups can form spontaneously if physical arrangements allow (for example, desks arranged in groups of four or more students working at tables).

Spontaneous helping between and among students as they work on problems individually can be encouraged.

For more structured projects, assign students to specific groups. It is usually a good idea to include a range of students and to have a strong reader in each group. Explain the problem or project and guide the groups as necessary. When students have finished, call a pair from each group to present and explain the results of their work or have the entire group present the results, with each member explaining one part of the solution or project. Having lower-performing students present first allows them to contribute, while higher-performing students expand on their efforts and give the fuller presentation.

Benefits: Students learn different strategies from each other for approaching a problem or task. They are invested in their classmates’ learning because the presentation will be on behalf of the whole group.
A Flexible Classroom

A Flexible Sense-Making
Math Talk Community

A sense-making Math Talk community involves flexible grouping because students are often moving between reasoning and discussing their reasoning. Many teachers also prefer to allow grouping structures to be mixed, with some students working alone, some in helping or collaborative pairs, and some in small groups. Helping or working together may arise spontaneously as needed. When using the whole-class version of Solve and Discuss, the class moves between problem solving and discussion. When introducing or discussing a topic, the discussion may stop while students think about something individually.

Student Collaboration

The collaboration and peer helping that is a central part of Math Expressions classrooms deepens students’ commitment to values such as responsibility and respect for others. As students reason about math situations or problems and collaboratively discuss their math thinking, they develop communication skills, sharpen their mathematical reasoning, and enhance their social awareness. Integrating students’ social and cultural worlds into their emerging math worlds helps them to find their own voices and to connect real world experiences to math concepts. With the teacher’s help, students’ collaborative capacities will grow over the year.

Student Leadership

Student leadership is an equally important part of the Math Expressions classroom. Student Leaders play many roles in Math Expressions classrooms. They lead many of the vital Quick Practice routines that start each math class. They can help other students during problem solving. They can manage and help with handing out MathBoards or other learning materials.

During whole class or small group Math Talk, students can use these responsive Math Talk, means of assistance to help others learn:

- engaging and involving
- managing
- coaching: modeling, cognitive restructuring and clarifying, instructing and explaining, questioning, feedback

A vital role of the teacher is to support students to develop as Student Leaders throughout the year.

Common Core Mathematical Practices

All eight of the Mathematical Practices relate to and support each other.

Connections Among Math Sense-Making, Structure, Drawings, and Talk Using All Mathematical Practices

Making sense (MP.1) of mathematical situations and symbols is the foundation of math teaching and learning. Math drawings (MP.4 and MP.5) that model situations or quantities help students see mathematical structure and generalize across quantities and situations (MP.7 and MP.8). Making math drawings and engaging in Math Talk about them is an interactive and continual sense-making process (all eight Mathematical Practices). Having a math drawing that everyone can see during Math Talk supports the sense-making of everyone (all eight Mathematical Practices). Working within a Math Talk learning community (MP.1 through MP.8) means that help is available. This supports students to preserve the problem structure in problem solving, an important part of MP.1.
Students are always encouraged to say things in their own words during Math Talk (MP.2 and MP.3). The teacher relates student sense-making language to the precise math terms, which helps students make sense of the formal math language. Gradually students understand and use the precise terms (MP.1 and MP.6). Students also can improve the precision of their math drawings and Math Talk (all eight Mathematical Practices). Teachers and other students help everyone along this path toward precision (MP.6).

Math drawings, equations, and the various steps used in paper and pencil problem solving are all tools that can be used strategically (MP.5). Modeling and using these tools (MP.4 and MP.5) with explanations (MP.3) help one’s own, as well as others’, learning process and sense-making (MP.1).

Math Expressions and the Common Core Standards
Many of the central features of the Common Core State Standards appeared in Math Expressions during its development and when it was first published. This is because Math Expressions drew on the same research base and international standards and programs used in the development of the CCSS. Therefore, the implementation of these central features of CCSS is well tested by the years of classroom development and by use in many classrooms in its published form. These are deeply and well developed in Math Expressions. A summary of these central features follows.

Math Drawings in the Common Core Standards
Math drawings (or diagrams) appear in standards in all domains in K–6 (OA, NBT, NF, MD, G). Students are encouraged to use math drawings to:

- represent and make sense of a situation or quantity
- relate the strategy to a written numerical method and explain the reasoning used (see below)

Students then transition to fluency with standard methods without drawings of quantities, but drawings of situations may continue to be needed and used for new or difficult problems.

The standards specify that drawings need not show details but should show the mathematics in the problem, i.e., they are math drawings. In later grades the CCSS use the term *diagram* instead of *drawing*.

Math Drawings are tools used in modeling (MP.4 and MP.5). They support sense-making, reasoning, and explaining (MP.1, MP.2, MP.3, MP.6). They require students to see structure and generalize (MP.7 and MP.8).

Reasoning Is Central in the Common Core Standards
Reasoning is explicitly and repeatedly mentioned in the standards as well as in the Mathematical Practices. Students are to reason about mathematical ideas. Importantly, such reasoning is supported by the visual/conceptual aspects of the standards discussed above and in the section on the Mathematical Practices. This enables age-appropriate learning paths to be developed by the teacher in the classroom.
Understanding and Fluency Are Both Crucial in the Common Core State Standards

Understanding and fluency are both mentioned repeatedly in various standards. Importantly, the standards are focused and coherent across grades so there is time to do both understanding and fluency. The Math Expressions learning path model of teaching and learning discussed earlier specifies how understanding and fluency can both be achieved.

Ambitious Algebraic Problems Appear in All Domains

The CCSS provide tables of core word problem types that describe different addition, subtraction, multiplication, and division situations. Importantly, any of the three quantities in each situation can be the unknown quantity. Some of these problem subtypes have a problem representation that differs from the solution representation or computation, so the original problem representation needs to be reflected on and re-represented. Students do this from Grade 1 on using informal reasoning rather than formal algebraic methods. But this process of representing the situation and then re-representing to find the solution is the same process used in algebra.

In Math Expressions students may do this representing with a situation equation, such as \( n + 4 = 9 \), and then re-represent this as a solution equation, \( 4 + n = 9 \) or \( 9 - 4 = n \). Or, students may represent situations with a Math Drawing or a numerical diagram into which they put numbers. Or, they may use both. Using situation and solution equations and/or diagrams as representations is research based. These mathematically desirable and accessible methods are an integral part of the Teaching Model described previously. Students solve these problem types using different numbers and measures as well as two-step and multistep problems.

Students solve but also make up word problems. As a result, they become comfortable and flexible with mathematical language and can connect concepts and terminology with meaningful referents from their own lives.

Seeing and Using Equations of Many Forms

Many students in this country struggle with algebra because they think equations have to have one number alone on the right. The CCSS emphasize that students need to see equations of different forms even in the early grades. Math Expressions does this beginning in Kindergarten. The first equation students see is of the form \( 5 = 4 + 1 \) to record decomposing 5 into two numbers. The equations and drawings record each decomposition. Students then discuss patterns across these decompositions. Students also write 8 related equations rather than just the usual 4 with one number on the right. They continue to work with many forms of equations as they represent and then re-represent problem situations.
Levels of Addition/Subtraction Single-Digit Solution Strategies

Operations and Algebraic Thinking (OA) standards specify a learning path of levels of addition/subtraction strategies from Kindergarten through Grade 2: direct model, count on, and convert to a simpler problem (make a ten, doubles plus or minus 1, and other derived fact methods). *Math Expressions* has always supported students to learn these methods as rapidly as possible. The second and third levels are the research-based, mathematically desirable and accessible methods. These methods are then extended to form a learning path of levels of multiplication and division methods.

*Math Expressions* and the CCSS emphasize the importance of understanding subtraction as an unknown-addend situation and division as an unknown-factor situation. This enables students to solve a wider range of problems and to develop easier methods that go forward rather than backward.

Research-Based Mathematically Desirable and Accessible Multidigit Methods

The Number and Operations in Base Ten (NBT) standards specify that students develop, discuss, and use efficient, accurate, and generalizable methods including the standard algorithm. Students are to:

- use concrete models or drawings and strategies based on place value and properties of operations
- relate the strategy to a written method and explain the reasoning used (explanations may be supported by drawings or objects)

*Math Expressions* has always approached multidigit computation in this way. The MathBoards are designed to support understanding of multidigit numbers. They help students learn to make math drawings for multidigit quantities. Students invent methods, and they see and discuss research-based mathematically desirable and accessible methods. These methods connect to how students think, and they stimulate discussions of crucial aspects of computation. Most of these methods involve minor variations of the current common ways of writing the standard algorithms but are simpler for students to understand and carry out. These methods can also be considered standard algorithms because algorithms concern the big ideas underlying them and not minor variations in how they are written.