From Current Research: Volume Measure

Cubes make especially nice units of volume because they are uniform and fit together snugly to make neat three-dimensional arrays. For this reason, people customarily use cubes as standard units of volume. Although students might work first with identical cubes of some other size, a cubic centimeter, 1 cm³, which is the volume of a 1-cm-by-1-cm-by-1-cm cube, is a natural first unit of volume to use. Alternatively, a cubic inch, 1 in³, could be used initially. The volume of a three-dimensional shape, in cubic units, is the number of 1-unit-by-1-unit-by-1-unit cubes (“unit cubes”) required to fill or make the shape without gaps or overlaps.


From Current Research: How Geometric Thinking Develops

At the visual level of thinking, figures are judged by their appearance. We say, “It is a square. I know that it is one because I see it is.” Children might say, “It is a rectangle because it looks like a box.”

At the next level, the descriptive level, figures are the bearers of their properties. A figure is no longer judged because “it looks like one” but rather because it has certain properties. For example, an equilateral triangle has such properties as three sides; all sides equal; three equal angles; and symmetry, both about a line and rotational. At this level, language is important for describing shapes. However, at the descriptive level, properties are not yet logically ordered, so a triangle with equal sides is not necessarily one with equal angles.
At the next level, the informal deduction level, properties are logically ordered. They are deduced from one another; one property precedes or follows from another property. Students use properties that they already know to formulate definitions—for example, for squares, rectangles, and equilateral triangles—and use them to justify relationships, such as explaining why all squares are rectangles or why the sum of the angle measures of any triangle must be 180°. However, at this level, the intrinsic meaning of deduction, that is, the role of axioms, definitions, theorems, and their converses, is not understood.

My experience as a teacher of geometry convinces me that all too often, students have not yet achieved this level of informal deduction. Consequently, they are not successful in their study of the kind of geometry that Euclid created, which involves formal deduction.


Other Useful Resources


Getting Ready to Teach Unit 8

Using the Common Core Standards for Mathematical Practice

The Common Core State Standards for Mathematical Content indicate what concepts, skills, and problem solving students should learn. The Common Core State Standards for Mathematical Practice indicate how students should demonstrate understanding. These Mathematical Practices are embedded directly into the Student and Teacher Editions for each unit in Math Expressions. As you use the teaching suggestions, you will automatically implement a teaching style that encourages students to demonstrate a thorough understanding of concepts, skills, and problems. In this program, Math Talk suggestions are a vehicle used to encourage discussion that supports all eight Mathematical Practices. See examples in Mathematical Practice 6.

**Mathematical Practice 1**

*Make sense of problems and persevere in solving them.*

Students analyze and make conjectures about how to solve a problem. They plan, monitor, and check their solutions. They determine if their answers are reasonable and can justify their reasoning.

**TEACHER EDITION: Examples from Unit 8**

**MP.1 Make Sense of Problems**

Problems 15 and 16 on Student Book page 242 require students to interpret the remainder. In Problem 15, the remainder is dropped because Sarita only wants full bottles. The remainder tells her how much she will have left over. In Problem 16, the quotient is increased by 1 because it will take 25 whole buckets and a partially filled bucket to fill the barrel.

**MP.1, MP.4 Make Sense of Problems/Model with Mathematics**

Problems 2 and 3 involve finding area and volume. Since no formulas are given, students should infer that \( A = l \cdot w \) is used to find the area of a rectangular face of the prism, and \( V = l \cdot w \cdot h \) is used to find the volume of the prism.

**Mathematical Practice 1** is integrated into Unit 8 in the following ways:

- Analyze Relationships
- Draw a Diagram
- Write a Formula
Mathematical Practice 2
Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. They can connect diagrams and equations for a given situation. Quantitative reasoning entails attending to the meaning of quantities. In this unit, this involves reasoning about the size of units when making conversions and conceptualizing volume.

**Common Core**

**MP.2 Reason Abstractly and Quantitatively**

Provide context for the examples to help students make sense of measurements and conversions.

- If you know a lawn is 15 yd long, how can you find its length in feet? *There are 3 ft in every one of the 15 yards, so multiply* $3 \times 15 = 45$ *ft.*

- When you convert from a large unit to a smaller unit, what operation do you use? *Why? Multiplication; you need more of the smaller units to measure the same length.*

- When you convert from a small unit to a larger unit, what operation do you use? *Why? Division; the number of units will decrease.*

- If you know a stick is 48 in. long, how can you find its length in feet? *There are 12 in. in a foot, so divide by 12; $48 \div 12 = 4$, so its length is 4 ft.*

**Mathematical Practice 2** is integrated into Unit 8 in the following ways:

**Connect Symbols and Words**

Problem 1 involves opposite faces of the aquarium. Be sure students understand that the opposite faces of a rectangular prism are congruent. In other words, the faces are identical in size and shape. Students applying this understanding will conclude that if the perimeter or area of one face of the prism is known, the perimeter or area of the opposite face is also known.

Lesson 4 Activity 1

Lesson 17 Activity 1
**Mathematical Practice 3**

**Construct viable arguments and critique the reasoning of others.**

Students use stated assumptions, definitions, and previously established results in constructing arguments. They are able to analyze situations and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Students are also able to distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Students can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Talk is a conversation tool by which students formulate ideas and analyze responses and engage in discourse. See also Mathematical Practice 6: Attend to Precision.

### TEACHER EDITION: Examples from Unit 8

**MP.3 Construct Viable Arguments**

**Compare Representations**

Students should be able to make these observations:

- Rectangle B has whole number side lengths. Rectangle C has fractional side lengths.
- Rectangle B contains 3 rows and 5 columns of square centimeters, so its area is 15 square centimeters.
- Rectangle C is one part of a square centimeter that is divided into 3 parts horizontally and 5 parts vertically. Its area is \( \frac{1}{15} \) square centimeter.
- For both rectangles, we can find the area by multiplying the length times the width.
- The area of Rectangle B is a whole number of square centimeters. The area of Rectangle C is a fraction of a square centimeter.

**What’s the Error?**

**Whole Class**

**MP.3, MP.6 Construct Viable Arguments/Critique Reasoning of Others**

**Puzzled Penguin**

Ask students to find quadrilateral L in their cards. Say:

- Puzzled Penguin put this in the oval for regular polygon because all the sides are congruent.
- Is this correct? no Why not? The angles are not congruent. To be a regular polygon, all the sides must be congruent and all the angles must be congruent.
- What does a regular polygon with four sides look like? a square

**Mathematical Practice 3** is integrated into Unit 8 in the following ways:

- Compare Representations
- Puzzled Penguin

Lesson 8 Activity 1
Mathematical Practice 4
Model with mathematics.

Students can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an equation to solve a problem. Students might draw diagrams to lead them to a solution for a problem. Students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation. They are able to identify important quantities in a practical situation and represent their relationships using such tools as diagrams, tables, graphs, and formulas.

MP.4, MP.5 Model with Mathematics/Use Appropriate Tools
Class MathBoard
Encourage students to suggest different kinds of numerical data that they could collect and show on a line plot. Two possible data sets are the number of pets each student has at home and the day of the month of each student's birthday. On the class MathBoard, a student volunteer can make a frequency table to summarize the data and draw a horizontal line with a scale for one of the ideas. Students can then go to the board and add their data to the table and the graph.

MP.4 Model with Mathematics
Build a Model
Students examine the concept of improvised units by designing a shipping box for boxes of tissue. Designs should reflect the idea that the dimensions should be multiples of the dimensions of the tissue box. For example, a height that is 3 times the height of the tissue box, a width that is 4 times the width of the tissue box, and a length that is 5 times the length of the tissue box.

• To order many shipping boxes for the tissues, what would you need to do to get a shipping box that holds 60 units? Bring the tissue box to show the size.
• Do you think it is easier to talk about volume using standard units rather than improvised units? Using standard units means everyone would understand the size of the unit.

Mathematical Practice 4 is integrated into Unit 8 in the following ways:

<table>
<thead>
<tr>
<th>Build a Model</th>
<th>Draw a Diagram</th>
<th>Write an Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centimeter Cubes</td>
<td>Make a Line Plot</td>
<td>Write an Expression</td>
</tr>
<tr>
<td>Class MathBoard</td>
<td>Write a Formula</td>
<td></td>
</tr>
</tbody>
</table>
Mathematical Practice 5

Use appropriate tools strategically.

Students consider the available tools and models when solving mathematical problems. Students make sound decisions about when each of these tools might be helpful. These tools might include paper and pencil, a straightedge, a ruler, or the MathBoard. They recognize both the insight to be gained from using the tool and the tool’s limitations. When making mathematical models, they are able to identify quantities in a practical situation and represent relationships using modeling tools such as diagrams, grid paper, tables, graphs, and equations.

Modeling numbers in problems and in computations is a central focus in Math Expressions lessons. Students learn and develop models to solve numerical problems and to model problem situations. Students continually use both kinds of modeling throughout the program.

**MP.4, MP.5 Model with Mathematics/Use Appropriate Tools**

**Centimeter Cubes** Allow students to work in Small Groups. The nets for Exercises 5–8 can be found on Student Activity Book page 258A or on Activity Workbook page 93. Students cut out each of the nets and form rectangular prisms. Explain to students that although rectangular prisms have 6 sides, these nets only have 5 to allow the prism to be open at the top so unit cubes can be placed inside.

After the students have folded and taped their nets, they fill each figure with unit cubes without any gaps or overlaps. They count the cubes and record the number on Student Book page 257. They should realize that the number of unit cubes is equal to the volume.

**MP.5 Use Appropriate Tools**

**MathBoard Modeling** On their MathBoards, students draw a rectangle that is 3 cm × 4 cm.

- Consider all the possible prisms that could have this base. What else do you need to know to build the prism or to know its volume? I need to know how many layers it has or its height.
- What is the area of the base? 12 sq cm
- Place one layer of cubes on the rectangle you drew. What is the volume of the prism? 12 cu cm
- How is the volume related to the base? The product of the area of the base times the height will give the volume of the prism.

Mathematical Practice 5 is integrated into Unit 8 in the following ways:

- **Build a Model**
- **Centimeter Cubes**
- **Class MathBoard**
- **Draw a Model**
- **MathBoard**
- **MathBoard Modeling**
- **Quadrilateral Cards**
- **Ruler**
Mathematical Practice 6: Attend to precision.

Students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. They are careful about specifying units of measure to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. Students give carefully formulated explanations to each other.

**Mathematical Practice 6 is integrated into Unit 8 in the following ways:**

- **Describe a Method**
- **Develop a Formula**
- **Explain an Example**
- **Puzzled Penguin**
- **Verify Solutions**

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**Teacher Edition: Examples from Unit 8**

**MP.6 Attend to Precision** Describe a Method

Focus students’ attentions on Exercise 7. Ask them how they could use the properties to find the volume of the prism mentally. Students should conclude that they can use the Associative Property to regroup the factors \((7 \times 5) \times 4\) to \(7 \times (5 \times 4)\). It is easy to multiply \(7 \times 20\) mentally.

Lesson 10 Activity 2

**MP.6 Attend to Precision** Verify Solutions

After most groups have finished, discuss the results. Draw each Venn diagram on the board or display and label a transparency of Venn Diagrams (TRB M60). Have a volunteer fill in the diagram. Discuss and resolve any disagreements students have.

Lesson 15 Activity 2

**Math Talk**

Use Solve and Discuss for Problem 7, reminding students that they should first write a situation equation and then a solution equation.

- **Situation equation:** \(V = 4 \times w \times 2 = 24\).
- **Solution equation:** \(24 \div (4 \times 2) = 24 \div 8 = 3 = w\)

Lesson 11 Activity 1

**Math Talk in Action**

This discussion helps students understand how to recognize hidden information in a problem.

What information does the problem give you and what information do you need that isn’t given to you?

- **Jane:** The problem tells us that all the scarves are 2.6 meters long.
- **Kyle:** We know that she makes a scarf every month for 2 years.
- **Tony:** We aren’t told how many months she knits, only how many years. Since I know that there are 12 months in a year, I can still work the problem.

Lesson 1 Activity 2
Mathematical Practice 7
Look for and make use of structure.

Students analyze problems to discern a pattern or structure. They draw conclusions about the structure of the relationships they have identified.

**TEACHER EDITION: Examples from Unit 8**

**MP.7 Look for Structure** Identify Relationships

If students are still struggling with metric conversions, encourage them to make a vertical list of the liquid volume units in order of size, with the largest at the top and smallest at the bottom, with an emphasis on the prefixes. Each step between units represents a conversion of one unit to the next larger or smaller unit. If the step is **up** one step, they divide by 1 power of ten ($\div 10$). If the step is **down** one step, they multiply by 1 power of ten ($\times 10$). If the unit change is two steps, they multiply or divide by 10 to the second power ($10^2$ or 100), three steps ($10^3$ or 1,000), etc.

- kiloliter $\div 10^3$
- hectoliter $\div 10^2$
- dekaliter $\div 10$
- liter $\times 10$
- deciliter $\times 10^2$
- centiliter $\times 10^3$
- milliliter

**MP.7 Look for Structure** Identify Relationships

Discuss relationships in the completed diagram.

- Which category has the most shapes? Why? Quadrilaterals; all the shapes have four straight sides, so they are all quadrilaterals.
- Shape E is in the rhombuses category and in the parallelograms and quadrilaterals categories. Why does this make sense? All rhombuses are parallelograms and all parallelograms are quadrilaterals.
- Let’s say you see right away that shape D belongs in the rectangles category. What other categories do you automatically know it belongs to? parallelograms and quadrilaterals
- Which shapes are in the most categories? B and R Why are they in so many categories? They are squares, and squares are also rectangles and rhombuses. And, rectangles and rhombuses are also parallelograms and quadrilaterals.
Mathematical Practice 8
Look for and express regularity in repeated reasoning.

Students use repeated reasoning as they analyze patterns, relationships, and calculations to generalize methods, rules, and shortcuts. As they work to solve a problem, students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

TEACHER EDITION: Examples from Unit 8

MP.8 Use Repeated Reasoning
Generalize a Formula For Exercise 7, point out that the figure is a square. Ask students if they can find another formula for finding the perimeter of a square. Students should determine that because all sides of a square are equal, they can use the formula \(4 \cdot s\), or \(s + s + s + s\), for finding the perimeter.

Mathematical Practice 8 is integrated into Unit 8 in the following ways:

- Draw Conclusions
- Generalize
- Generalize a Formula

FOCUS on Mathematical Practices
Unit 8 includes a special lesson that involves solving real world problems and incorporates all 8 Mathematical Practices. In this lesson, students use what they know about formulas to calculate the perimeter, area, and volume of aquariums.

STUDENT EDITION: LESSON 17, PAGES 273–274

Math and Aquariums
A graphical level is a small aquarium. Other aquaria, like those found in stores, can be enormous and have computer monitored and controlled life-support systems. Many home aquariums are made of glass or acrylic, and are shaped like rectangular prisms. A sketch of Naomi’s home aquarium is shown at right.

Use the sketch to solve Problems 1 and 2.

1. Which faces of Naomi’s aquarium—top and bottom, the sides, or the front and back—are the greatest perimeter?
2. Use a formula and find the area of each of the following faces.
   - top and bottom
   - sides
   - front and back

3. Use a formula and find the volume of the aquarium. 18” by 12” by 15”

4. Suppose Naomi would like to attach rubber edging along the top edges, and along the bottom edges, of her aquarium. Use a formula to determine the minimum length of edging Naomi would need.

5. Naomi would like to place a flat sheet of acrylic under her aquarium, with the sheet extending 1 inch beyond the edges of the aquarium in all directions. What size sheet of acrylic should she purchase? 17 inches by 32 inches

6. Suppose Naomi would like to place a flat sheet of acrylic under her aquarium, with the sheet extending 1 inch beyond the edges of the aquarium in all directions. What size sheet of acrylic should she purchase? 17 inches by 32 inches

7. Use the sketch of Naomi’s aquarium shown below to solve Problems 8-10.

8. Suppose three inches of sand were placed under her aquarium, with the sheet extending 1 inch beyond the edges of the aquarium in all directions. What size sheet of acrylic should she purchase? 17 inches by 32 inches

9. Suppose Naomi’s aquarium is composed of six identical rectangular prisms each measuring \((12 \times 18 \times 8)\), and use the computation \(6 \times (12 \times 18 \times 8)\) to calculate its volume.
Getting Ready to Teach Unit 8

Learning Path in the Common Core Standards

In Grade 4, students used multiplication to convert larger units to smaller units within the same measurement system. In this unit, students use both multiplication and division to convert within the same system. This is the first grade level in which students convert smaller units to larger units.

Students worked with the perimeter and area of rectangles in earlier grades. At this grade level, they work with the key concept of volume, exploring the concept using hands-on unit cubes, and progressing in their work to using a formula.

Students also have previous experience identifying geometric figures by their properties. In Grade 5, students draw as well as sort and classify polygons by their attributes. They begin to formulate the idea of a hierarchy of quadrilateral properties.

Visual models and real world situations are used throughout the unit to illustrate important concepts.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- **Lesson 1**: Not doubling length and width when finding perimeter of a rectangle
- **Lesson 9**: Counting only visible unit cubes when counting is used to find volume
- **Lesson 16**: Not recognizing characteristics of polygons

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.
**Converting Measurements**

**Metric Units of Measure** Generally speaking, it is simpler to convert one metric unit of measure to another than it is to convert one customary unit of measure to another. Converting customary units requires multiplying or dividing by a wide variety of numbers. Converting metric units requires only multiplying or dividing by a power of 10. Powers of 10 include $10^1$ or 10, $10^2$ or 100, $10^3$ or 1,000, and so on.

Multiplying or dividing by a power of 10 produces a result that is the same as shifting the digits in the measurement a number of places to the left or the right.

### Dividing by:

- $10^1$ shifts the digits 1 place to the right.
- $10^2$ shifts the digits 2 places to the right.
- $10^3$ shifts the digits 3 places to the right.

And so on.

### Multiplying by:

- $10^1$ shifts the digits 1 place to the left.
- $10^2$ shifts the digits 2 places to the left.
- $10^3$ shifts the digits 3 places to the left.

And so on.

**Generalizations** When completing the activities in Lessons 1–3, students should develop an understanding of the following generalizations for converting metric units of measure.

- Multiplication is used to convert a larger unit to a smaller unit.
- Division is used to convert a smaller unit to a larger unit.

Once students become familiar with the relationships that metric units of measure share (i.e., How are meters related to kilometers and millimeters?), they often can perform a variety of metric conversions using only mental math.
**Metric Units of Length** The metric units of length students work with and convert in this unit include millimeters (mm), centimeters (cm), decimeters (dm), meters (m), dekameters (dam), hectometers (hm), and kilometers (km).

In the metric system, the meter is the basic unit of length. The relationships shown below—comparing 1 meter to other metric units of length and 1 of other metric units of length to meters—are used by students to perform conversions.

<table>
<thead>
<tr>
<th>Metric Units of Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 dekameter (dam) = 10 meters</td>
</tr>
<tr>
<td>1 hectometer (hm) = 100 meters</td>
</tr>
<tr>
<td>1 kilometer (km) = 1,000 meters</td>
</tr>
<tr>
<td>1 meter = 10 decimeters (dm)</td>
</tr>
<tr>
<td>1 meter = 100 centimeters (cm)</td>
</tr>
<tr>
<td>1 meter = 1,000 millimeters (mm)</td>
</tr>
</tbody>
</table>

Initially, students use guided examples to convert, and progress to conversions with no guidance.

**Example 1** Convert to a Smaller Unit

2 km = ____ m

Step 1: Choose multiplication because we will need more of the smaller units.

Step 2: Multiply by 1,000 because 1,000 m = 1 km.

2 km = 2,000 m (2 \times 1,000 = 2,000)

**Example 2** Convert to a Larger Unit

50 cm = ____ m

Step 1: Choose division because we will need fewer of the larger units.

Step 2: Divide by 100 because 100 cm = 1 m.

50 cm = 0.5 m (50 ÷ 100 = 0.5)

**Complete.**

1. 15 m = ____ mm  
2. 877 cm = ____ m  
3. 450 m = ____ km  
4. 2.39 m = ____ cm  
5. 2,040 mm = ____ m  
6. 8.6 km = ____ m
Two-Step and Multistep Problems Exercises and problems in Lesson 1 range from straightforward conversions (such as those shown on the previous page) to multistep problems in real world contexts.

14. Natasha ran 3.1 kilometers. Tonya ran 4 meters more than half as far as Natasha. How many meters did Tonya run? 1,554 meters

24. Mattie is making a collar for her dog. She needs to buy some chain, a clasp, and a name tag. She wants the chain to be 40 centimeters long. A meter of the chain costs $9.75. The clasp is $1.29, and the name tag is $3.43. How much will it cost to make the collar? Estimate to check if your answer is reasonable.

$8.62; Estimate: \( \frac{1}{2} \times 10 \) + $1 + $3 = $9

Liquid Volume and Mass As with length, the concepts of liquid volume and mass in Lessons 2 and 3 are introduced with charts that show the relationships between the basic units, liters and grams, and the other units of liquid volume and mass.

### Metric Units of Liquid Volume

<table>
<thead>
<tr>
<th>Metric Units of Liquid Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 dekaliter (daL) = 10 liters</td>
</tr>
<tr>
<td>1 hectoliter (hL) = 100 liters</td>
</tr>
<tr>
<td>1 kiloliter (kL) = 1,000 liters</td>
</tr>
<tr>
<td>1 liter = 10 deciliters (dL)</td>
</tr>
<tr>
<td>1 liter = 100 centiliters (cL)</td>
</tr>
<tr>
<td>1 liter = 1,000 milliliters (mL)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Metric Units of Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 dekagram (dag) = 10 grams</td>
</tr>
<tr>
<td>1 hectogram (hg) = 100 grams</td>
</tr>
<tr>
<td>1 kilogram (kg) = 1,000 grams</td>
</tr>
<tr>
<td>1 gram = 10 decigrams (dg)</td>
</tr>
<tr>
<td>1 gram = 100 centigrams (cg)</td>
</tr>
<tr>
<td>1 gram = 1,000 milligrams (mg)</td>
</tr>
</tbody>
</table>

For these concepts, students again use guided examples to convert, and progress to conversions with no guidance and then to solving multistep problems in real world contexts.
**Customary Units of Measure** Although the strategies of using multiplication to change to a smaller unit and using division to change to a larger unit are the same for both metric and customary conversions, computations for customary conversions are more complicated because they do not involve powers of 10. In other words, performing customary conversions is not as simple as shifting digits to the left or to the right. For example, changing millimeters (the smallest metric unit of length) to kilometers (the largest unit) simply involves shifting the digits six places to the right (i.e., dividing by $10^6$ or 1,000,000). The related customary conversion of inches to miles would typically involve first dividing by 12 to find the number of feet, then dividing by 5,280 to change the number of feet to miles. Metric conversions by comparison are very straightforward.

**Length, Liquid Volume, and Weight** To successfully convert customary units of length, liquid volume, and weight in Lessons 4–6, students must know a wide range of customary relationships.

<table>
<thead>
<tr>
<th>Customary Units of Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 foot (ft) = 12 inches (in.)</td>
</tr>
<tr>
<td>1 yard (yd) = 3 feet = 36 inches</td>
</tr>
<tr>
<td>1 mile (mi) = 1,760 yards = 5,280 feet</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Customary Units of Liquid Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 gallon (gal) = 4 quarts (qt) = 8 pints (pt) = 16 cups (c)</td>
</tr>
<tr>
<td>$\frac{1}{4}$ gallon = 1 quart = 2 pints = 4 cups</td>
</tr>
<tr>
<td>$\frac{1}{8}$ gallon = $\frac{1}{2}$ quart = 1 pint = 2 cups</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Customary Units of Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pound (lb) = 16 ounces (oz)</td>
</tr>
<tr>
<td>1 ton (T) = 2,000 pounds</td>
</tr>
</tbody>
</table>
As with metric conversions, students initially use guided examples to convert customary units.

**Example 1 Convert to a Smaller Unit**

15 yd = ____ ft

Step 1: Choose multiplication because we will need more of the smaller units.
Step 2: Multiply by 3 because 3 ft = 1 yd.

15 yd = 45 ft (15 × 3 = 45)

**Example 2 Convert to a Larger Unit**

104 pt = ____ gal

Step 1: Choose division because we will need fewer of the larger units.
Step 2: Divide by 8 because 8 pt = 1 gal

104 pt = 13 gal (104 ÷ 8 = 13)

**Complete.**

1. \( \frac{1}{2} \) yd = ____ ft
2. ____ T = 14,000 lb
3. ____ oz = 5\( \frac{1}{4} \) lb
4. ____ yd = 144 in.
5. 2\( \frac{1}{2} \) qt = ____ pt
6. ____ gal = 48 qt

7. What fraction of 1 quart is 1 cup?

8. What fraction of 1 gallon is 3 pints?

**Two-Step and Multistep Problems** Word problems related to customary length, liquid volume, and weight in Lessons 4–6 include real world contexts.

11. A \( \frac{1}{4} \)-lb package of sunflower seeds costs 79¢. An 8-ounce package costs $1.59. Which package represents the lower cost per ounce? 

   the \( \frac{1}{4} \)-lb package

16. A serving size for a glass of pineapple-orange punch is \( \frac{1}{2} \) cup. Liam needs to make 72 servings of punch. He will use 8 pints of pineapple juice. The rest is orange juice. How many pints of orange juice does he need to make the punch?

   10 pints
Fractions Conversions involving customary measurements often include fractional units. For example, the height of a wall is more likely to be stated as $8\frac{1}{2}$ feet instead of 8 feet 6 inches, and a distance jogged is more likely to be stated as $2\frac{1}{4}$ miles instead of 2.25 miles, which in spoken form is *two and twenty-five hundredths miles*. Students working with customary measurements and converting customary units need an understanding of how to find products and quotients when the computations include fractions.

Mass vs. Weight Although mass and weight are different concepts, the terms are often used interchangeably in general, everyday usage. The terms are used properly when *mass* refers to a measurement of the amount of matter an object contains, and *weight* refers to a measurement of the pull of gravity on an object.

A way for students to understand this difference is to present the terms in a real world context. For example, a bowling ball here on Earth has a given mass and weight; suppose that its weight is 12 pounds. If placed on a scale on the moon, where the force of gravity is about $\frac{1}{6}$ that of Earth, the bowling ball would have a weight of about $\frac{1}{6}$ of 12, or 2 pounds. However, in both locations, the mass, or amount of matter that makes up the ball, is unchanged. Stated a different way, weight changes with location; mass does not.

Number Sense Using both metric and customary measures in real world contexts enables students to help make sense of the measures and better understand the relationships that the measures share.
Line Plots

A line plot is a measure of frequency. Stated a different way, a line plot is a visual display that shows how often something occurs.

The symbols of a line plot typically include dots or X's although students can use any mark to record data. The symbols used in Math Expressions are dots. A key concept for students to understand is that the symbols display a 1-to-1 relationship to frequency. For example, a line plot showing the number of votes each candidate in a class election received would display the same number of symbols as the number of students that voted.

The horizontal axis of a line plot can display whole numbers or fractions, as shown below. Using given sets of data, students complete both types of plots in Lesson 7. The number of symbols above each whole number or fraction is the measure of frequency.

6. For 10 days, Mario measured the amount of food that his cat Toby ate each day. The amounts Mario recorded are shown in the table at the right. Graph the results on the line plot.

Students’ work with line plots also requires them to interpret the data.

a. What is the total amount of food Toby ate over the 10 days? Explain how you got your answer.

b. What amount of food would Toby get if it were distributed evenly each day over the 10 days?
Perimeter and Area

When students use the length and the width of a rectangle to calculate its perimeter or area, they sometimes don’t know which dimension represents the length and which dimension represents the width. Given the rectangle at the right, for example, some students may assume that 45 mm, or the side-to-side measure, is the width of the rectangle, while others may generalize that 15 mm represents the width because the length of a rectangle must always be greater than its width.

Because addition and multiplication are commutative (i.e., the order of the addends or factors does not change the sum or product), the length and width measures of a rectangle can be used interchangeably when finding perimeter or area. However, there is some consensus in the math community that length is the longer dimension of an object.

**Conceptual Understanding**  Tick marks and unit squares help students build a conceptual understanding of perimeter and area.

![Perimeter and Area Diagram]

Formula: \( P = 2l + 2w \)

Formula: \( A = l \times w \)

This conceptual understanding helps when students work with fractional side lengths.

![Fractional Area Diagram]

\[ A = \frac{1}{3} \text{ cm} \times \frac{1}{5} \text{ cm} = \frac{1}{15} \text{ sq cm} \]

\[ A = \frac{2}{3} \text{ cm} \times \frac{4}{5} \text{ cm} = \frac{8}{15} \text{ sq cm} \]
**Volume**

**Cubic Units** Nets are used to explore the concept of volume and introduce students to the idea of three-dimensional thinking. Students tape and fold each net so that one face remains open. Then they fill the net with unit cubes and count the cubes to find the volume of the prism.

**Volume Concepts** As students explore volume, they discuss these volume concepts.

- A cube with side length 1 unit is called a “unit cube.”
- A solid figure that can be packed without gaps or overlaps using \( n \) unit cubes has a volume of \( n \) cubic units.
- The same-sized cubes must be used to determine volume.

**Volume Formula** The hands-on activity above, along with a variety of exercises (such as the one shown below), enable students to begin to think in terms of layers—a prerequisite skill for deriving the formula \( V = l \cdot w \cdot h \) used for finding the volume of a prism.

Each layer of these rectangular prisms is 4 cubes by 2 cubes. How many cubes make up each prism?

- 1 layer: 8 cubes
- 2 layers: 16 cubes
- 3 layers: 24 cubes
- 4 layers: 32 cubes
- 5 layers: 40 cubes
- 6 layers: 48 cubes
Relate Perimeter, Area, and Volume  The exercises in Lesson 12 help students compare and contrast perimeter, area, and volume.

Tell if you need to measure for length, area, or volume. Then write the number of measurements you need to make.

1. How tall are you?   **length; 1**

2. How much carpet is needed for a floor?  **area; 2**

3. How much sand is in a sandbox?  **volume; 3**

4. How much wallpaper is needed for one wall?  **area; 2**

5. How long is a string?  **length; 1**

6. How much space is there inside a refrigerator?  **volume; 3**

Lesson 12 also gives students an opportunity to apply their understanding of perimeter, area, and volume in real world contexts.

9. Soledad has a storage box. The box is $6\frac{1}{2}$ inches long, $4\frac{3}{4}$ inches wide, and 7 inches tall. She wants to run a border around the top of the box. How much border does she need?

   **$22\frac{1}{2}$ in.**

10. The refrigerator is $5\frac{2}{3}$ feet tall, $2\frac{2}{7}$ feet wide, and $2\frac{1}{4}$ feet deep. How much space does the refrigerator take up on the floor?

   **$5\frac{1}{7}$ sq ft**

11. Melissa is stacking storage cubes in a crate. The bottom of the crate is 8 inches by 12 inches. The volume of the crate is 768 cubic inches. If a storage cube has a length of 4 inches, how many storage cubes will fit in a crate?

   **12 storage cubes**
Composite Solid Figures In Lesson 13, students extend their understanding of finding volume to finding the volume of composite solids made of two or more rectangular prisms. This involves decomposing the composite figures. Initially, colors are used to help students see the individual sections that are part of the composite solid. Example 1 and Example 2 illustrate two possible ways to decompose the solid. After finding the volume, students verify that the volume is the same no matter how they break the figure apart.

Students progress to finding the volume of composite figures that do not use shading to aid decomposition, and they solve real world problems.

Find the volume of each composite figure.

4. \[ V = 1,632 \text{ cubic centimeters} \]

9. An in-ground swimming pool often has steps that are made from poured concrete. In the sketch of the steps at the right, the steps are identical, each measuring 18 inches from side to side, 12 inches from front to back, and 8 inches tall.

Calculate the amount of concrete that is needed to form the steps.

10,368 cubic inches
**Two-Dimensional Figures**

**Attributes of Quadrilaterals** Lesson 14 provides opportunities to build reasoning and classifying skills with respect to quadrilaterals.

Write *true* or *false*. If the statement is false, sketch a counterexample.

*Sketches will vary. Samples are given.*

1. All quadrilaterals have at least one pair of parallel sides.

   - **false**

   - **true**

2. All squares have adjacent sides that are perpendicular.

- **true**

**Sketch a shape that fits the description if possible.**

*Sketches will vary. Samples are given.*

7. A parallelogram with exactly two right angles

   - **not possible**

8. A trapezoid with one line of symmetry

   - **not possible**

Students also study how the categories of quadrilaterals are related, sorting shapes into each category (or multiple categories) and then completing statements about the categories.
Attributes of Triangles  The activities in Lesson 15 extend reasoning and classifying skills to include triangles.

3. All right triangles have two acute angles.  
   true

4. Any triangle with an obtuse angle must be scalene.  
   false

8. a triangle with two right angles  
   not possible

9. a triangle with more than one line of symmetry  

Students also study how the categories of triangles are related, sorting shapes in diagrams.
Attributes of Other Two-Dimensional Shapes   The activities in Lesson 16 extend reasoning and classifying skills to include a variety of polygons.

1. not a polygon; not closed

3. polygon

5. not a polygon; not made from segments

7. triangle

8. pentagon

9. octagon

10. hexagon

Focus on Mathematical Practices

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about perimeter, area, and volume to solve problems related to aquariums.
Building a Math Talk Community

**MATH TALK** Frequent opportunities for students to explain their mathematical thinking strengthen the learning community of your classroom. As students actively question, listen, and express ideas, they increase their mathematical knowledge and take on more responsibility for learning. Use the following types of questions as you build a Math Talk community in your classroom.

**Elicit student thinking**
- So, what is this problem about?
- Tell us what you see.
- Tell us your thinking.

**Support student thinking**
- What did you mean when you said _____?
- What were you thinking when you decided to _____?
- Show us on your drawing what you mean.
- Use wait time: Take your time.... We’ll wait....

**Extend student thinking**
- Restate: So you’re saying that _____.
- Now that you have solved the problem in that way, can you think of another way to work on this problem?
- How is your way of solving like _____’s way?
- How is your way of solving different from _____’s way?

**Increase participation of other students in the conversation**
- Prompt students for further participation: Would someone like to add on?
- Ask students to restate someone else’s reasoning: Can you repeat what _____ just said in your own words?
- Ask students to apply their own reasoning to someone else’s reasoning:
  - Do you agree or disagree, and why?
  - Did anyone think of this problem in a different way?
  - Does anyone have the same answer, but got it in a different way?
  - Does anyone have a different answer? Will you explain your solution to us?

**Probe specific math topics:**
- What would happen if _____?
- How can we check to be sure that this is a correct answer?
- Is that true for all cases?
- What pattern do you see here?