Putting Research into Practice

From Our Curriculum Research Project
Dividing by Decimals

Decimals less than 1 The most difficult aspect of dividing by decimals is that dividing by a decimal less than 1 gives a result that is greater than the divided number. So for multiplying and dividing by a decimal less than 1, the shift in the place value of the result is opposite to the shift when multiplying and dividing by whole numbers. As with multiplying, students need strong visual and conceptual supports to understand this new result for dividing by decimals. We found that we could use what they had learned from multiplying by decimals as a basis for understanding the patterns for dividing by decimals. Dividing by 0.1 is asking how many small parts (tenths) are in each place of the number being divided. Students can see that the resulting pattern shifts each place to the left: it gets one place larger. Dividing by 0.01 shifts a number two places to the left (it gets two places larger). This reasoning applies to decimal factors as well as whole number factors, so students now have a general method. They can look at specific examples to understand, and use the rule about moving the decimal place to the right (the number gets larger) as many decimal places as are in the divisor. And they can see that multiplying the dividend and divisor by some form of 1 will result in the same answer (the count places method).

Is it multiplication or division? Students then need to solve mixed problems involving multiplication and division, predicting and explaining whether the result will increase or decrease and why. They need repeated experiences differentiating multiplication and division situations because they cannot tell these by the size of the numbers as they could with whole numbers. They need to focus on the structure of the problem situations.
From Current Research  
Types of Problems Solved by Division

**Equal groups situations** In grades 3 and 4 students solve a variety of problems for each of the two types of equal-groups division situations:

- “How many in each group?” or partitive, division situations, and
- “How many groups?” or measurement, division situations.

**“How many in each group?” division problems** Suppose 24 students will form 3 equal groups. How many students will be in each group? This is a “how many in each group?” situation that can be shown as

\[ 24 \div 3 = ? \]

Students can also view this problem as an “unknown factor” multiplication problem, namely as “3 times how many equals 24?” or

\[ 3 \times ? = 24 \]

which connects division and multiplication.

**“How many groups?” division problems** Suppose 24 students will work in groups of 3. How many groups will there be? This is a “how many groups?” situation that can be shown as

\[ 24 \div 3 = ? \]

Again students can view this problem as an “unknown factor” multiplication problem, namely as “how many groups of 3 make 24?” or

\[ ? \times 3 = 24 \]

This might also be asked, “What number times 3 equals 24?” As before, viewing a division problem as a “missing-factor” problem connects multiplication and division.


Other Useful References


Getting Ready to Teach Unit 5

Using the Common Core Standards for Mathematical Practice

The Common Core State Standards for Mathematical Content indicate what concepts, skills, and problem solving students should learn. The Common Core State Standards for Mathematical Practice indicate how students should demonstrate understanding. These Mathematical Practices are embedded directly into the Student and Teacher Editions for each unit in Math Expressions. As you use the teaching suggestions, you will automatically implement a teaching style that encourages students to demonstrate a thorough understanding of concepts, skills, and problems. In this program, Math Talk suggestions are a vehicle used to encourage discussion that supports all eight Mathematical Practices. See examples in Mathematical Practice 6.

**COMMON CORE**

**Mathematical Practice 1**

**Make sense of problems and persevere in solving them.**

Students analyze and make conjectures about how to solve a problem. They plan, monitor, and check their solutions. They determine if their answers are reasonable and can justify their reasoning.

**TEACHER EDITION: Examples from Unit 5**

**MP.1 Make Sense of Problems** Analyze Problems If students have trouble deciding what to do with the remainder, ask them if a sensible answer to the problem is a whole number. If so, then they must decide whether to drop the remainder or round up, or whether the remainder is the answer. The gardener in Problem 7 cannot make just part of a trip, so it is necessary to round up. Pablo, in Problem 9, cannot buy part of a marker, so the remainder is dropped. In Problem 11, the remainder is the answer.

**Mathematical Practice 1** is integrated into Unit 5 in the following ways:

- Act it Out
- Analyze the Problem
- Reasonable Answers
- Look for a Pattern
- Use a Diagram
- Solve a Simpler Problem

**Lesson 4 ACTIVITY 2**

**MP.1 Make Sense of Problems** Look for a Pattern Complete Exercises 1–12 as a class. Discuss the patterns that occur when we find the cost of a single marble in dimes (divide $0.312 by 0.1), in pennies (divide $0.312 by 0.01), and in tenths of a cent (divide $0.312 by 0.001).

**Lesson 8 ACTIVITY 1**
**Mathematical Practice 2**  
**Reason abstractly and quantitatively.**

Students make sense of quantities and their relationships in problem situations. They can connect diagrams and equations for a given situation. Quantitative reasoning entails attending to the meaning of quantities. In this unit, this involves reasoning about the size of quotients using estimation, rounding, and mental math, and understanding how to interpret remainders in real world contexts.

**TEACHER EDITION: Examples from Unit 5**

**MP.2 Reason Quantitatively**  
Ask students to look at page 153 of the Student Book. Discuss what happened in the Digit-by-Digit method. In this division, standard rounding procedures (rounding 85 to 90) produce an estimated digit that is too small. Elicit from the students how we know that the digit is too small. When we multiply and subtract, the result is greater than the divisor. That tells us that we can form at least one more group.

**Lesson 3 Activity 1**

**MP.2 Reason Abstractly and Quantitatively**  
Ask students to look back at the quotients for $6 ÷ 0.2$, $6 ÷ 0.02$, and $6 ÷ 0.002$.

- As the divisor gets smaller—that is, as it goes from 0.2 to 0.02 to 0.002—what happens to the quotient? It gets larger. It goes from 30 to 300 to 3,000.
- Can you explain why? The smaller the parts we divide a whole number into, the more parts there will be.

**Lesson 7 Activity 3**

**Mathematical Practice 2** is integrated into Unit 5 in the following ways:

- Reason Abstractly
- Reason Abstractly and Quantitatively
- Reason Quantitatively
ACTIVITY 2

UNIT 5 MATH BACKGROUND

Mathematical Practice 3
Construct viable arguments and critique the reasoning of others.

Students use stated assumptions, definitions, and previously established results in constructing arguments. They are able to analyze situations and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Students are also able to distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Students can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Talk is a conversation tool by which students formulate ideas and analyze responses and engage in discourse. See also Mathematical Practice 6: Attend to Precision.

**TEACHER EDITION: Examples from Unit 5**

**MP.3 Construct Viable Arguments**

**Justify Conclusions**
Exercises 10–17 give students the opportunity to apply the ideas from Problems 6–9. Emphasize that they should not do the computations. They should instead reason about the numbers. For example, on the left side of Exercise 10, we are multiplying 356 by a whole number greater than 1, so the result will be greater than 356. On the right side, we are dividing 356 by a whole number greater than 1, so the result will be less than 356. Therefore, the left side is greater.

**What’s the Error?**

**Whole Class**

**MP.3, MP.6 Construct Viable Arguments/Critique Reasoning of Others**

Puzzled Penguin
Ask the class to read Puzzled Penguin’s letter on Student Book page 168. Many of your students have probably figured out that dividing by one tenth is the same as multiplying by ten, and found a similar correspondence for dividing by one hundredth. When we divide by one tenth, we are asking, “How many tenths are there in the number?” The answer is the same as if we multiplied by ten.

**Mathematical Practice 3** is integrated into Unit 5 in the following ways:

- Puzzled Penguin
- Justify Conclusions
**COMMON CORE**

**Mathematical Practice 4**

**Model with mathematics.**

Students can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an equation to solve a problem. Students might draw diagrams to lead them to a solution for a problem. Students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation. They are able to identify important quantities in a practical situation and represent their relationships using such tools as diagrams, tables, graphs, and formulas.

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**TEACHER EDITION: Examples from Unit 5**

**MP.4 Model with Mathematics** Write this problem on the board.

An airplane travels the same distance every day. It travels 3,822 miles in a week. How far does the airplane travel each day?

Ask a volunteer to draw a rectangle model on the board. Point out that the rectangle model represents the problem and corresponds to any solution method.

Invite several students to work at the board to solve the problem using any method. Discuss the various strategies for solving the problem.

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**MP.1, MP.4 Make Sense of Problems/Model with Mathematics** Use a Diagram

Ask students to read the text about transporting sheep on the top of Student Book page 151. Briefly discuss the rectangle model. We know the product (the area) and one of the factors (one side length). We must divide to find the other factor (the other side length).

Students may visualize this rectangle as an array of unit squares, as shown below. They can imagine that each square contains a sheep. The 32 unit squares in each column represent the 32 sheep in a train car. How many columns can we form with 2,048 sheep?

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**Mathematical Practice 4** is integrated into Unit 5 in the following ways:

- Class MathBoard
- Use a Diagram
- Play Money
Mathematical Practice 5

Use appropriate tools strategically.

Students consider the available tools and models when solving mathematical problems. Students make sound decisions about when each of these tools might be helpful. These tools might include paper and pencil, a straightedge, a ruler, or the MathBoard. They recognize both the insight to be gained from using the tool and the tool’s limitations. When making mathematical models, they are able to identify quantities in a practical situation and represent relationships using modeling tools such as diagrams, grid paper, tables, graphs, and equations.

Modeling numbers in problems and in computations is a central focus in Math Expressions lessons. Students learn and develop models to solve numerical problems and to model problem situations. Students continually use both kinds of modeling throughout the program.

TEACHER EDITION: Examples from Unit 5

MP.4, MP.5 Model with Mathematics/Use Appropriate Tools

Play Money

Organize students into groups of three and distribute the play money. Give each group at least 2 ten-dollar bills, 20 one-dollar bills, 22 dimes, and 15 pennies. Ask students to make $20.25 using 2 ten-dollar bills, 2 dimes, and 5 pennies.

- I want you to find a way to share this money equally among the three of you.
- To divide the money, you will need to trade some bills and coins for smaller bills and coins.
- Make notes about what you do, so you can explain it to the class.

Lesson 6 ACTIVITY 1

MathBoard

On the Class MathBoard, invite a volunteer to model the simpler problem above by drawing 5 circles to represent the dollars, using “p” to represent one peso, and then distributing 60 “p’s” equally among the 5 dollars. Ask the volunteer to explain the process, and have the seated students duplicate the actions on their individual MathBoards.

Lead students to conclude that the division $20.25 \div 3$ is used to find the value of 1 dollar, and then have them complete Problem 1.

Lesson 11 ACTIVITY 1

Mathematical Practice 5 is integrated into Unit 5 in the following ways:

Class MathBoard
Play Money
Mathematical Practice 6
Attend to precision.

Students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. They are careful about specifying units of measure to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. Students give carefully formulated explanations to each other.

MP.6 Attend to Precision Verify Solutions
For each exercise, ask a volunteer to check the solution. You may need to remind students that, when the solution has a remainder, we check the answer by multiplying the divisor by the quotient and then adding the remainder. If the answer is correct, the result will be the dividend.

Problem 39 Divide the cost for all 12 balls, $23.64, by 12 to get the cost per ball, $1.97. To check for reasonableness, use numbers that are close to the actual numbers but that are easy to compute with: $24 ÷ 12 = $2. The balls should cost about $2 each, so $1.97 is reasonable.

Problem 41 Multiply 4.5 miles per hour times 0.75 hour to get 3.375 miles. This is reasonable because the answer should be a little more than 4 × 0.75, which is 3.

MP.6 Attend to Precision Explain Solutions
Here are some possible methods for solving and checking the problems:

Problem 39 Divide the cost for all 12 balls, $23.64, by 12 to get the cost per ball, $1.97. To check for reasonableness, use numbers that are close to the actual numbers but that are easy to compute with: $24 ÷ 12 = $2. The balls should cost about $2 each, so $1.97 is reasonable.

Problem 41 Multiply 4.5 miles per hour times 0.75 hour to get 3.375 miles. This is reasonable because the answer should be a little more than 4 × 0.75, which is 3.

Mathematical Practice 6 is integrated into Unit 5 in the following ways:

- Describe a Method
- Describe Methods
- Explain a Solution
- Explain Solutions
- Puzzled Penguin
- Verify Solutions
Mathematical Practice 7
Look for and make use of structure.

Students analyze problems to discern a pattern or structure. They draw conclusions about the structure of the relationships they have identified.

**TEACHER EDITION: Examples from Unit 5**

**MP.7 Look for Structure**  Identify Relationships  Review the fact that multiplication and division are inverse operations. Write $40 ÷ 5 = 8$ on the board. Ask a volunteer to write an equation that shows the inverse operation and checks the answer to the division problem. $5 \cdot 8 = 40$ or $8 \cdot 5 = 40$

**MP.7 Use Structure**  Write Exercise 1 on the board.

- What digits will be in the quotient? 4 and 9 How do you know? This problem has the same digits as the problem in the directions.

Write 49 above the 15 in the dividend.

- How can we figure out where to put the decimal point? The divisor is a whole number, so we don’t need to move any decimal points. The decimal point goes above the decimal point in the dividend.

Students should complete Exercises 2–8. If students have difficulty, suggest that they start by writing 49 above 15 in the dividend. They can then focus on placing the decimal point.

Mathematical Practice 7 is integrated into Unit 5 in the following ways:

- Identify Relationships
When you travel from one country to another, you sometimes need to exchange your currency for the currency used in the country you are visiting. An exchange rate is the rate at which one currency can be exchanged for another. Currencies are usually compared to 1 U.S. dollar (1 USD) when they are exchanged. For example, 1 USD may be exchanged for 6.5 Chinese yuans or 0.95 Canadian dollars. The exact amount of the exchange often varies from day to day.

Solve.

1. Suppose 5 U.S. dollars (5 USD) can be exchanged for 64 Mexican pesos. What operation would be used to find the value of 1 USD in pesos?

\[ \text{1 USD} = \frac{64 \text{ pesos}}{5 \text{ USD}} \]

2. Complete the exchange rate column of the table.

<table>
<thead>
<tr>
<th>Country</th>
<th>Currency Unit</th>
<th>Equivalent Amount</th>
<th>Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>yen</td>
<td>20 USD = 1,530 yen</td>
<td>1 USD = 76.5 yen</td>
</tr>
<tr>
<td>England</td>
<td>pound</td>
<td>10 USD = 6.1 pounds</td>
<td>1 USD = 0.163 pounds</td>
</tr>
<tr>
<td>Germany</td>
<td>Euro</td>
<td>50 USD = 35 Euros</td>
<td>1 USD = 0.7 Euros</td>
</tr>
</tbody>
</table>

Visiting another country often means exchanging more than 1 USD for the currency of that country.

3. The exchange rate for francs, the currency of Switzerland, is 10 USD = 8.8 francs. At that rate, how many francs would be exchanged for 25 USD?

\[ 25 \text{ USD} \times \frac{8.8 \text{ francs}}{10 \text{ USD}} = 22 \text{ francs} \]

4. A traveler in Latvia exchanged 5 USD for 2.6 lats. At that rate, what is the cost of a souvenir in lats if the cost is 3 USD?

\[ 3 \text{ USD} \times \frac{2.6 \text{ lats}}{5 \text{ USD}} = 1.62 \text{ lats} \]

5. The cost to visit a famous tourist attraction in Russia is 381.25 rubles. What is the cost in USD if the exchange rate is 3 USD = 91.5 rubles?

\[ 381.25 \text{ rubles} \times \frac{3 \text{ USD}}{91.5 \text{ rubles}} = 12.50 \text{ USD} \]
Getting Ready to Teach Unit 5

Learning Path in the Common Core Standards

In Grade 4, students learned to divide multidigit numbers by one-digit divisors. In this unit, students build on what they learned in Grade 4 as they explore dividing multidigit numbers by two-digit divisors.

Students also learn a strategy for changing a division problem with a decimal divisor to an equivalent problem with a whole number divisor. They solve real world division problems and interpret the remainders in the contexts of the problems. They also use estimation to determine whether answers are reasonable.

Real world situations are used throughout the unit to illustrate important division concepts.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- **Lesson 1:** Stopping division too soon, without considering the 0 in the ones place
- **Lesson 4:** Incorrectly interpreting the remainder of a real world division problem
- **Lesson 7:** Shifting the digits in the wrong direction when dividing by 0.1 and 0.01
- **Lesson 8:** Misplacing the decimal point in a quotient

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.
Methods of Multidigit Division

Division Methods In the first several lessons, students explore dividing multidigit whole numbers by one- and two-digit numbers. Students discuss and compare three division methods: Place Value Sections, Expanded Notation, and Digit-by-Digit.

An airplane travels the same distance every day. It travels 3,822 miles in a week. How far does the airplane travel each day?

**Place Value Sections**

<table>
<thead>
<tr>
<th>Digit</th>
<th>500</th>
<th>40</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3,822</td>
<td>322</td>
<td>546</td>
</tr>
<tr>
<td>3,500</td>
<td>280</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>322</td>
<td>42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Expanded Notation**

<table>
<thead>
<tr>
<th>Digit</th>
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<td>42</td>
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</tbody>
</table>

**Digit-by-Digit**

<table>
<thead>
<tr>
<th>Digit</th>
<th>54</th>
<th>6</th>
</tr>
</thead>
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<td>280</td>
<td>42</td>
</tr>
<tr>
<td>32</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

Students see that because dividing by a two-digit number requires using estimation, it is easy to overestimate or underestimate a digit of a quotient. They discuss how to adjust estimates when this occurs.

**Reasonable Answers** Students learn ways to use estimation to determine whether an answer to a division problem is reasonable.

“I know that $1,200 \div 6$ is 200 and $1,800 \div 6$ is 300. Because $1,350$ is between $1,200$ and $1,800$, my answer should be between 200 and 300. It is.”
Students explore the different ways a remainder can be interpreted.

- **The remainder may be dropped or ignored.**

  A roll of ribbon is 1,780 inches long. It takes 1 yard of ribbon (36 inches) to wrap a gift. How many gifts can be wrapped?

  \[ 1,780 \div 36 = 49 \text{ R}16 \]
  
  The answer is 49 gifts. The remainder is dropped because 16 inches is not enough to wrap a gift.

- **The answer may be rounded up to the next whole number.**

  There are 247 people traveling to the basketball tournament by bus. Each bus holds 52 people. How many buses will be needed?

  \[ 247 \div 52 = 4 \text{ R}39 \]
  
  This represents 4 full buses and 39 extra people. Another bus will be needed for 39 people, so the answer is 5.

- **The remainder may be used to form a fraction.**

  The 28 students in Mrs. Colby’s class will share 98 slices of pizza equally. How many slices will each student get?

  \[ 98 \div 28 = 3 \text{ R}14 \]
  
  This means each student gets 3 slices and there are 14 slices left. These leftover slices can be divided, giving each student an additional \(\frac{14}{28}\) slice, or \(\frac{1}{2}\) slice. The answer is, therefore, 3\(\frac{1}{2}\) slices.

- **The answer may be given as a decimal number.**

  Suppose 16 friends earned $348 at a car wash. They want to divide the money equally. How much should each friend get?

  \[ 348 \div 16 = 21 \text{ R}12 \]
  
  Each friend gets $21 and there are $12 left. Dividing the $12 gives each friend an additional \(\frac{12}{16}\) dollar, which is \(\frac{3}{4}\) dollar, or $0.75. The answer is $21.75.

- **The remainder may be the answer.**

  A bagel shop has 138 bagels to be packed into boxes of 12 to be sold. The extra bagels are for the workers. How many bagels will the workers get?

  \[ 138 \div 12 = 11 \text{ R}6 \]
  
  The bags are packed into 11 boxes. The workers get the 6 extra bagels, so the answer is 6 bagels.
Dividing a Decimal Number by a Whole Number

Concrete Models In Lesson 6, students divide decimal numbers by one- and two-digit whole numbers. They begin the lesson by using play money to model a division situation. Then they see how their modeling translates to a numerical solution.

Division Methods Students find that the same division methods they use for dividing a whole number by a whole number work for dividing a decimal by a whole number. They also observe that just as when we divide a whole number by a whole number, any leftover amount from one place is ungrouped and moved to the next place.

Three friends set up a lemonade stand and made $20.25. They will share the money equally. Study the steps below to see how much money each person should get.

When the $20 is split 3 ways, each person gets $6. There is $2 left.

\[ \begin{array}{c}
6 \\
3 \overline{)20.25} \\
-18 \\
\hline
2.2
\end{array} \]

We change the $2 to 20 dimes and add the other 2 dimes. There are 22 dimes.

\[ \begin{array}{c}
6.\overline{7} \\
3 \overline{)20.25} \\
-18 \\
\hline
2.2 \\
-2.1 \\
\hline
.1
\end{array} \]

When we split 22 dimes 3 ways, each person gets 7 dimes. There is 1 dime left.

We change the dime to 10 cents and add the other 5 cents. Now we split 15 cents 3 ways.

\[ \begin{array}{c}
6.75 \\
\color{red}{3} \overline{)20.25} \\
\color{red}{-18} \\
\hline
\color{red}{2.2} \\
\color{red}{-2.1} \\
\hline
\color{red}{.15}
\end{array} \]
Dividing by a Decimal Number

Shift Patterns  Students begin Lesson 7 by using the concept of money to explore how the digits in a whole number shift when that number is divided by 0.1, 0.01, or 0.001. In Lesson 8, students do a similar exploration with dividing decimal numbers. In both cases, they find the following:

- Dividing a number by 0.1 means finding the number of tenths in that number. Because there are 10 tenths in every whole, dividing by 0.1 is the same as multiplying by 10. For example, \(45.6 \div 0.1 = 45.6 \times 10 = 456\).
- Dividing a number by 0.01 means finding the number of hundredths in that number. Because there are 100 hundredths in every whole, dividing by 0.01 is the same as multiplying by 100. For example, \(45.6 \div 0.01 = 45.6 \times 100 = 4,560\).
- Dividing a number by 0.001 means finding the number of thousandths in that number. Because there are 1,000 thousandths in every whole, dividing by 0.001 is the same as multiplying by 1,000. For example, \(45.6 \div 0.001 = 45.6 \times 1,000 = 45,600\).

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Dividing by 0.1 and 0.01  As with decimal multiplication, students can first examine the cases of dividing by 0.1 and 0.01 to see that the quotient becomes 10 times or 100 times as large as the dividend.
Connect to Fractions  In Lessons 7 and 8, students learn a strategy for dividing by a decimal. They multiply both the dividend and the divisor by a power of 10 (that is, 10, 100, 1,000, and so on) to change the problem to an equivalent problem with a whole number divisor. They know this method works because it is equivalent to rewriting a fraction as an equivalent fraction.

You can use the strategy below to change a division problem with a decimal divisor to an equivalent problem with a whole number divisor.

**Discuss each step used to find 6 ÷ 0.2.**

**Step 1:** Write 6 ÷ 0.2 as a fraction. 

\[
6 \div 0.2 = \frac{6}{0.2}
\]

**Step 2:** Make an equivalent fraction with a whole number divisor by multiplying \( \frac{6}{0.2} \) by 1 in the form of \( \frac{10}{10} \). Now you can divide 60 by 2.

\[
\frac{6 \times 10}{0.2 \times 10} = \frac{60}{2}
\]

21. Why is the answer to 60 ÷ 2 the same as the answer to 6 ÷ 0.2? 

The fractions \( \frac{60}{2} \) and \( \frac{6}{0.2} \) are equivalent, so the division problems 60 ÷ 2 and 6 ÷ 0.2 must also be equivalent.

Students see that they do not need to rewrite the division problem in fraction form to use this method. They can simply move the decimal point in both numbers the same number of places.

You can use the strategy of multiplying both numbers by 10 even when a division problem is given in long division format.

**Step 1:** Put a decimal point after the whole number. 

\[
0.2 \div 6.
\]

**Step 2:** Multiply both numbers by 10, which shifts the digits one place left. Show this by moving the decimal point one place right. Add zeros if necessary.

\[
0.2 \times 10 \div 6.0 = 3.0
\]

**Step 3:** Instead of drawing arrows, you can make little marks called carets (^) to show where you put the “new” decimal points. Now divide 60 by 2.

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*from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN*

**Division by a Decimal** Students can then proceed to more general cases. For example, to calculate 7 ÷ 0.2, students can reason that 0.2 is 2 tenths and 7 is 70 tenths, so asking how many 2 tenths are in 7 is the same as asking how many 2 tenths are in 70 tenths. In other words, 7 ÷ 0.2 is the same as 70 ÷ 2; multiplying both the 7 and the 0.2 by 10 results in the same quotient.
Compare and Contrast Multiplication and Division

Choosing the Correct Operation  In Lesson 10, students start by solving word problems involving multiplication and division with decimals. They must carefully consider the context of each problem to determine which operation to use.

1. A turtle walks 0.2 mile in 1 hour. How far can it walk in 0.5 hour?
   a. Do you need to multiply or divide to solve? ______ multiply ______
   b. Will the answer be more or less than 0.2 miles? ______ less ______
   c. What is the answer? ______ 0.1 mile ______

2. Gus ran 3.6 miles. He took a sip of water every 0.9 mile. How many sips did he take?
   a. Do you need to multiply or divide to solve? ______ divide ______
   b. Will the answer be greater or less than 3.6? ______ greater ______
   c. What is the answer? ______ 4 sips ______

Making Generalizations  Students make generalizations about multiplying and dividing a whole number by another number. Specifically, students discuss the following points:

► Multiplying a whole number a by a whole number greater than 1 gives a product greater than a.
► Multiplying a whole number a by a decimal less than 1 gives a product less than a.
► Dividing a whole number a by a whole number greater than 1 gives a quotient less than a.
► Dividing a whole number a by a decimal less than 1 gives a quotient greater than a.

Dividing by a Decimal  Students can summarize the results of their reasoning as specific numerical patterns then as one general overall pattern such as “when the decimal point in the divisor is moved to make a whole number, the decimal point in the dividend should be moved the same number of places.”
Predict the Relative Size of the Answer Students solve mixed multiplication and division word problems in which they must first predict the size of the answer relative to the size of one of the numbers in the problem. This requires them to reason about the operation that is required and about the results of multiplying or dividing by a number greater than or less than 1.

18. Farmer Ortigoza has 124.6 acres of land. Farmer Ruben has 0.8 times as much land as Farmer Ortigoza.
   a. Does Farmer Ruben have more or less than 124.6 acres?
      __________ less
   b. How many acres does Farmer Ruben have? __________ 99.68 acres

19. Mee Young has 48 meters of crepe paper. She will cut it into strips that are each 0.6 meter long.
   a. Will Mee Young get more or fewer than 48 strips?
      __________ more
   b. How many strips will Mee Young get? __________ 80 strips

Focus on Mathematical Practices

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use decimal operations to compute currency exchange rates.