From Our Curriculum Research Project

The most difficult aspect of multiplying by decimals is that multiplying by a decimal less than 1 gives a result that is less than the multiplied number. This is new for students because up to now their experience with whole number multiplication leads them to expect a larger result. Students need strong visual and conceptual supports to understand this new result. We found that it was powerful to use what students know about multiplying by 10 and 100 to find out what happens when they multiply by 0.1 and 0.01. By thinking about numbers place by place, they see that multiplying by 10 shifts each place to the left: it gets one place bigger. But multiplying by one-tenth (0.1) shifts each place to the right: it gets one place smaller as they find one-tenth of each place. Multiplying by 100 shifts a number two places to the left (two places larger), and multiplying by 0.01 shifts a number two places to the right (two places smaller).

This reasoning applies to decimal factors as well as to whole number factors, so students now have a general method. They can look at specific examples to understand and use the rule that the product will have the number of decimal places that is the sum of the places in the factors. Often this rule is only a rote rule about moving the decimal point, and it can easily be forgotten or confused. Our students had a foundation of reasoning about shifting the multiplied number to get larger or smaller, so they could use this understanding to check answers and remember the rule.
From Current Research

Multidigit Multiplication

Researchers have reported a preliminary learning progression of multidigit methods for third- to fifth-grade classrooms in which teachers fostered children’s invention of algorithms (Baek 1998). These methods moved from (a) direct modeling with objects or drawings (such as by ones and by tens and ones), to (b) written methods involving repeatedly adding—sometimes by repeated doubling, a surprisingly effective method used historically, to (c) partitioning methods. The partitioning methods ranged from partitioning using numbers other than 10, partitioning one number into tens and ones, and partitioning both numbers into tens and ones.


Deepening Understanding

Ultimately, the meanings of decimal numbers and multiplication, as well as the concepts developed in mental computation, are used as tools by the students to discover generalizations, communicate their ideas about decimal multiplication, and justify why rules work. By engaging in these processes, students have opportunities to build connections to other mathematical topics (such as proportional reasoning) and to other content areas (science and social studies). In forming these links, students learn mathematical content with deeper understanding.


Other Useful References


Getting Ready to Teach Unit 4

Using the Common Core Standards for Mathematical Practice

The Common Core State Standards for Mathematical Content indicate what concepts, skills, and problem solving students should learn. The Common Core State Standards for Mathematical Practice indicate how students should demonstrate understanding. These Mathematical Practices are embedded directly into the Student and Teacher Editions for each unit in *Math Expressions*. As you use the teaching suggestions, you will automatically implement a teaching style that encourages students to demonstrate a thorough understanding of concepts, skills, and problems. In this program, Math Talk suggestions are a vehicle used to encourage discussion that supports all eight Mathematical Practices. See examples in Mathematical Practice 6.

**Mathematical Practice 1**

*Make sense of problems and persevere in solving them.*

Students analyze and make conjectures about how to solve a problem. They plan, monitor, and check their solutions. They determine if their answers are reasonable and can justify their reasoning.

**TEACHER EDITION: Examples from Unit 4**

**MP.1 Make Sense of Problems**

*Act It Out*  
Ask three student volunteers to go to the board and enact the shift patterns that will occur when $325$ is multiplied by $10$. Begin by first writing 6 large line segments for the place values, and then put a comma in the middle. Write 3, 2, and 5 in the appropriate places. Assign one student to be the 3 hundreds, another to be the 2 tens, and the other to be the 5 ones. Ask the students at the board to shift positions to show how $325$ changes when it is multiplied by 10 and to explain their shift as they do it. Write the numbers in their new places above the old numbers.

**Lesson 1 ACTIVITY 1**

**MP.1 Make Sense of Problems**

*Reasonable Answers*  
Encourage Small Groups of students to discuss how they can decide whether or not their answers are reasonable. Possible methods might include:

- **Problem 21** Students can recognize that their answer should be between $4 \times 25 = 100$ and $5 \times 25 = 125$.
- **Problem 22** A reasonable answer would be close to the product of $3 \times 3$.
- **Problem 23** Rounding the factors to 2 and 70 would predict an answer that was close to 140 pounds.

**Lesson 11 ACTIVITY 2**

**Mathematical Practice 1** is integrated into Unit 4 in the following ways:

<table>
<thead>
<tr>
<th>Act it Out</th>
<th>Check Answers</th>
<th>Look for a Pattern</th>
<th>Reasonable Answers</th>
</tr>
</thead>
</table>
COMMON CORE

Mathematical Practice 2

Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. They can connect diagrams and equations for a given situation. Quantitative reasoning entails attending to the meaning of quantities. In this unit, this involves reasoning about shift patterns to determine how multiplying by tens or multiplying by decimals will affect the product.

TEACHER EDITION: Examples from Unit 4

MP.2 Reason Abstractly and Quantitatively Connect Symbols and Words As Student Pairs focus on Student Book page 107, they should discuss the explanation of exponential form. One partner should then answer the odd numbered exercises and the other partner should answer the even numbered exercises. They can then check each other’s work and discuss any discrepancies.

Remind students that they explored shift patterns using exponents in Lesson 1. Review with them that the exponent is equal to the number of zeros to the right of the 1 (10¹ = 10, 10² = 100, 10³ = 1,000).

Write the following problems on the board and have students identify which way the digits shift. Then have them write the exponential form as repeated multiplication and solve.

14 × 10¹ left one space, 14 × 10 = 140
14 × 10² left two spaces, 14 × 10 × 10 = 1,400
14 × 10³ left three spaces, 14 × 10 × 10 × 10 = 14,000

Mathematical Practice 2 is integrated into Unit 4 in the following ways:

Connect Diagrams and Equations
Connect Symbols and Words
**Mathematical Practice 3**

*Construct viable arguments and critique the reasoning of others.*

Students use stated assumptions, definitions, and previously established results in constructing arguments. They are able to analyze situations and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Students are also able to distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Students can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Talk is a conversation tool by which students formulate ideas and analyze responses and engage in discourse. See also Mathematical Practice 6: Attend to Precision.

---

**MP.3 Construct Viable Arguments**

**Compare Methods** On page 115 of the Student Book, Small Groups of students can compare all of the multiplication methods. Use Exercises 7 and 8 to discuss the similarities and differences among the methods.

Elicit from the class that Place Value Sections and Expanded Notation are set up so that each place is multiplied by each place, while Place Value Rows and the Short Cut method are set up so that the entire top number is multiplied by each place in the multiplier.

---

**What’s the Error?**

**MP.3, MP.6 Construct Viable Arguments/Critique Reasoning of Others**

Puzzled Penguin On the board, write the three dollar amounts and the way Puzzled Penguin rounded them:

- $1.35 \approx $1.4
- $0.07 \approx $0.1
- $12.39 \approx $12.4

- Is Puzzled Penguin correct?
- What did Puzzled Penguin do wrong? Puzzled Penguin did not write the cents to the hundredths place.
- How can we round to the nearest tenth and still write the dollar amount correctly?

Elicit from students that to write the dollar amount for ten cents, they should write $0.10. Students should identify that 0.1 and 0.10 are two possible ways to write one tenth.

---

**Mathematical Practice 3** is integrated into Unit 4 in the following ways:

- Compare Methods
- Puzzled Penguin
**Mathematical Practice 4**  
*Model with mathematics.*

Students can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an equation to solve a problem. Students might draw diagrams to lead them to a solution for a problem. Students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation. They are able to identify important quantities in a practical situation and represent their relationships using such tools as diagrams, tables, graphs, and formulas.

**MP.4 Model with Mathematics**  
Ask the class to solve Problem 2 on Student Book page 111. Draw a few large rectangles on the board and invite several student volunteers to work there. One model is shown below.

```
39 × 54
30 × 50 = 1,500
30 × 4 = 120
9 × 50 = 450
9 × 4 = 36
30 × 50 = 1,500
30 × 4 = 120
9 × 50 = 450
9 × 4 = 36

2,106
```

**Concrete Objects**  
Read the first problem on Student Book page 119 together. As volunteers explain the calculations done in each line of the chart, have them act out the multiplication by using play money. Students can see that modeling 3 groups of 9 cents is the same as writing $3 × 0.09$. Discuss the answers to Problems 1 and 2.

The problem about Mia is more complex, but can be solved by multiplying each place value by each place value, just as with whole numbers. These basic kinds of multiplications are similar to the first problem, except now the results are combined. Expanded Notation allows students to see each partial product for each place value. When multiplying by 20, students use the shift pattern for multiplying by 10 discussed in Lesson 1.

**Lesson 3**  
**ACTIVITY 1**

**Lesson 6**  
**ACTIVITY 1**

**Mathematical Practice 4** is integrated into Unit 4 in the following ways:

- Class MathBoard
- Concrete Objects
- Write an Equation
Mathematical Practice 5

Use appropriate tools strategically.

Students consider the available tools and models when solving mathematical problems. Students make sound decisions about when each of these tools might be helpful. These tools might include paper and pencil, a straightedge, a ruler, or the MathBoard. They recognize both the insight to be gained from using the tool and the tool’s limitations. When making mathematical models, they are able to identify quantities in a practical situation and represent relationships using modeling tools such as diagrams, grid paper, tables, graphs, and equations.

Modeling numbers in problems and in computations is a central focus in *Math Expressions* lessons. Students learn and develop models to solve numerical problems and to model problem situations. Students continually use both kinds of modeling throughout the program.

**TEACHER EDITION: Examples from Unit 4**

**MP.4, MP.5 Model with Mathematics/Use Appropriate Tools**

Concrete Objects

Ask two student volunteers to be **Student Leaders** who use play money to act out these multiplication problems. The rest of the class can form **Small Groups** and follow along, drawing the money on their MathBoards.

The first Student Leader shows a 1-dollar bill. The second Student Leader says and shows 1 dollar multiplied by 10. (ten 1-dollar bills) Student Leader 1 writes $10 on the board.

Then, Student Leader 1 shows a 10-dollar bill. Student Leader 2 says and shows $10 multiplied by 10. (ten 10-dollar bills) Student Leader 1 writes $100 on the board.

Discuss how the digits shift to the left every time they are multiplied by 10. The 1 keeps shifting another place to the left. ($1, $10, $100)

Class MathBoard

Exercise 5 on Student Book page 142, provides an opportunity to model the connection shared by multiplication and repeated addition. Invite one student to sketch six confused cloudwings on the Class MathBoard while the other students sketch at their desks. After each butterfly’s wingspan is labeled in inches (1.26), have students write and complete the repeated addition and the multiplication that can be used to find the combined length of all six wingspans.

\[
\begin{align*}
1.26 + 1.26 + 1.26 + 1.26 + 1.26 + 1.26 &= 7.56 \\
1.26 \times 6 &= 7.56
\end{align*}
\]

Point out that multiplication is a fast and efficient way to perform repeated addition.

**Mathematical Practice 5** is integrated into Unit 4 in the following ways:

Concrete Objects

Class MathBoard
**COMMON CORE Mathematical Practice 6**

**Attend to precision.**

Students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. They are careful about specifying units of measure to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. Students give carefully formulated explanations to each other.

**TEACHER EDITION: Examples from Unit 4**

**MP.6 Attend to Precision** Explain a Method  Ask a volunteer to explain the Expanded Notation method. Each factor is separated into its tens and ones and then all 4 pair combinations formed by the tens and ones are multiplied to produce the 4 partial products.

Ask another volunteer to compare variations A and B. Variation A shows the 4 multiplication equations that give the partial products; variation B shows only the partial products.

**Mathematical Practice 6** is integrated into Unit 4 in the following ways:

<table>
<thead>
<tr>
<th>Describe a Method</th>
<th>Explain a Solution</th>
<th>Identify Relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explain a Method</td>
<td>Explain Solutions</td>
<td>Puzzled Penguin</td>
</tr>
</tbody>
</table>
Mathematical Practice 7
Look for and make use of structure.

Students analyze problems to discern a pattern or structure. They draw conclusions about the structure of the relationships they have identified.

### TEACHER EDITION: Examples from Unit 4

**MP.7 Look for Structure** Identify Relationships

You may want to point to the 5-row on the Class Multiplication Table Poster to review the five-pattern there. Or write these problems on the board and have students give the answers quickly. They will be reminded that every other answer ends in zero because it is a multiple of 10. (Even numbers of 5s make multiples of 10.)

- \(5 \times 1 = 5\)
- \(5 \times 2 = 10\)
- \(5 \times 3 = 15\)
- \(5 \times 4 = 20\)
- \(5 \times 5 = 25\)
- \(5 \times 6 = 30\)
- \(5 \times 7 = 35\)
- \(5 \times 8 = 40\)

**MP.7 Look for Structure** Identify Relationships

Encourage students to work in pairs to solve problems that include a whole number and a decimal. Have Student Pairs use pencil and paper to multiply 294 by 0.1 and by 0.01. Provide 3 more whole numbers to multiply by 0.1 and by 0.01. Discuss the pattern that students notice. Students should be able to see that the digits in the whole number are moved over 1 and 2 places to the right when they multiply by 0.1 and by 0.01, respectively.

Mathematical Practice 7 is integrated into Unit 4 in the following ways:
Identify Relationships
**ACTIVITY 3**

21/02/12  1:45 PM  Planning  
21/02/12  1:46 PM  

---

### Focus

wingspans of various insects. with decimals to compare the 
what they know about operating 
Practices. In this lesson, students use 
and incorporates all 8 Mathematical 
involves solving real world problems 
Unit 4 includes a special lesson that 
reasonableness of their intermediate results.

---

### TEACHER EDITION: Examples from Unit 4

**MP.8 Use Repeated Reasoning**

**Generalize** Discuss the key idea by asking students to determine the pattern in the number of decimal places in the factors and in the product. When you multiply a decimal number by a whole number, the product has the same number of decimal places as the decimal factor.

Ask students to consider how the key idea can assist them as they multiply by decimals.

**Mathematical Practice 8** is integrated into Unit 4 in the following ways:

- **Conclude**
- **Draw Conclusions**
- **Generalize**

---

### STUDENT EDITION: LESSON 12, PAGES 141–142

**MP.8 Use Repeated Reasoning**

**Draw Conclusions** Focus students’ attention on Student Book page 136. The multiplications in Exercises 29–31 and 33–35 might appear to be exceptions to the generalization. However, as students perform the multiplications, they will see that they are not exceptions. Review the idea about where zeros do and do not change the value of a decimal number.

---

**FOCUS on Mathematical Practices**

Unit 4 includes a special lesson that involves solving real world problems and incorporates all 8 Mathematical Practices. In this lesson, students use what they know about operating with decimals to compare the wingspans of various insects.
Getting Ready to Teach Unit 4

Learning Path in the Common Core Standards
In this unit, students multiply whole numbers and decimals. Their work involves finding the products of two-digit whole numbers and decimals both less than 1 and greater than 1, and estimating products by rounding factors.

Visual models and real world situations are used throughout the unit to illustrate important number and operation concepts.

Help Students Avoid Common Errors
Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- Lesson 2: Assuming the number of zeros in a product is always the same as the number of zeros in the factors
- Lesson 4: Regrouping incorrectly when finding a product
- Lesson 7: Shifting the digits in the wrong direction
- Lesson 8: Unable to apply properties to simplify finding products
- Lesson 9: Not using zero as a placeholder and using an exponent as a factor
- Lesson 10: Rounding cents incorrectly

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.
Shift Patterns in Multiplication

Multiplying Whole Numbers by Powers of Ten  Real world scenarios involving money are used to introduce students to the concept of shifting digits. For whole numbers, Jordan’s salary is given as $243 per week, and students consider the amount earned in 1, 10, 100, and 1,000 weeks.

### Jordan’s Weekly Earnings

\[
\begin{align*}
\text{Week 1} & : \quad \$243 \\
\text{Week 10} & : \quad 10 \times \$243 = \$2430 \\
\text{Week 100} & : \quad 10 \times 10 \times \$243 = \$24300 \\
\text{Week 1000} & : \quad 10 \times 10 \times 10 \times \$243 = \$243000
\end{align*}
\]

After 10 Weeks

\[
\begin{align*}
\text{Weeks} & : \quad \$2430 \\
\times 10 & : \quad \$24300
\end{align*}
\]

After 100 Weeks

\[
\begin{align*}
\text{Weeks} & : \quad \$24300 \\
\times 100 & : \quad \$243000
\end{align*}
\]

### Jordan’s Earnings

Jordan’s earnings show students that the result of multiplying by 1, 10, 100 and 1,000 shifts the digits in the multiplicand to the left. Multiplying by 10 shifts the digits one place to the left, by 100 shifts the digits two places, and by 1,000 shifts the digits three places.

---

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

**Place Value and Shift Patterns**

Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths.

New at Grade 5 is the use of whole number exponents to denote powers of 10. Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. For example, multiplying by \(10^4\) is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit 4 places to the left. Patterns in the number of 0s in products of whole numbers and a power of 10, and the location of the decimal point in products of decimals with powers of 10, can be explained in terms of place value.
Multiplying Decimals by Powers of Ten  For decimals, the same concept is presented by giving the manufacturing cost of an item as $0.412, and having students consider the cost for the production of 1, 10, 100, and 1,000 items.

### Cost of a Red Phantom Marble

\[
\begin{align*}
\text{Cost of a Red Phantom Marble} & \\
\$ & \underline{0} \underline{4} \underline{1} \underline{2} \times 1 & = & \underline{0} \underline{4} \underline{1} \underline{2} \\
1 \times \$0.412 & = & $0.412 \\
\end{align*}
\]

### 10 Red Phantom Marbles

\[
\begin{align*}
\text{10 Red Phantom Marbles} & \\
\$ & \underline{0} \underline{4} \underline{1} \underline{2} & \times 10 & = & \underline{0} \underline{4} \underline{1} \underline{2} \\
10 \times \$0.412 & = & $4.12 \\
\end{align*}
\]

### 100 Red Phantom Marbles

\[
\begin{align*}
\text{100 Red Phantom Marbles} & \\
\$ & \underline{4} \underline{1} \underline{2} \underline{0} & \times 100 & = & \underline{4} \underline{1} \underline{2} \underline{0} \\
100 \times \$0.412 & = & $41.20 \\
\end{align*}
\]

### 1,000 Red Phantom Marbles

\[
\begin{align*}
\text{1,000 Red Phantom Marbles} & \\
\$ & \underline{4} \underline{1} \underline{2} \underline{0} \underline{0} & \times 1,000 & = & \underline{4} \underline{1} \underline{2} \underline{0} \underline{0} \\
1,000 \times \$0.412 & = & $412.00 \\
\end{align*}
\]

Again students see that multiplying by 10, 100, and 1,000 shifts the digits in the multiplicand to the left. Multiplying by 10 shifts the digits one place to the left, by 100 shifts the digits two places, and by 1,000 shifts the digits three places.
Patterns in Multiplying with Zeros Exploring the Shift Pattern of zeros in the table below enables students to generalize that shifting digits any number of places to the left shifts the decimal point the same number of places to the right. It also gives students a way to predict the number of zeros in a product.

<table>
<thead>
<tr>
<th>×</th>
<th>3</th>
<th>30</th>
<th>300</th>
<th>3,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>a. 2 × 3</td>
<td>2 × 30</td>
<td>2 × 300</td>
<td>2 × 3,000</td>
</tr>
<tr>
<td></td>
<td>= 6</td>
<td>= 6 × 10</td>
<td>= 6 × 100</td>
<td>= 6 × 1,000</td>
</tr>
</tbody>
</table>

Powers of 10 Students work with expanded and exponential forms of powers of 10. This enables students to connect factors of 10, 100, and 1,000 to repeated multiplication and exponents.

- \( 10 = 10 \times 1 \) exponential form: \( 10^1 \)
- \( 100 = 10 \times 10 \) exponential form: \( 10^2 \)
- \( 1,000 = 10 \times 10 \times 10 \) exponential form: \( 10^3 \)

This concept is extended to multiplying with powers of 10.

\[
5 \times 10 \times 10 = 5 \times 10^2 \\
3 \times 10^3 = 3 \times 10 \times 10 \times 10
\]

Patterns With Fives and Zeros In Lesson 2, students discover that using a pattern of zeros to predict the number of zeros in a product requires special attention for some combinations of factors. For example, the product \( 2 \times 50 \) contains one zero in the factors but two zeros in the product.

- \( \text{the product of } 2 \times 5 \text{ is } 10 \) \( \text{2} \times 50 = 100 \)

Lesson 2 makes students aware that using a pattern of zeros (e.g., counting zeros in the factors) must be done with care when predicting the reasonableness of a product.

Generalizations The real world examples in Lessons 1 and 2 lead students to the following place value generalizations.

- Multiplying by 10 produces a number 10 times as great and shifts each digit one place to the left.
- Multiplying by 100 produces a number 100 times as great and shifts each digit two places to the left.
- Multiplying by 1,000 produces a number 1,000 times as great and shifts each digit three places to the left.
**Multidigit Multiplication**

In these lessons students discuss, analyze, and draw models for contexts involving multiplication of two-digit numbers, and work with methods that can be used to find the products.

**Place Value Sections** One way to model the multiplication of two-digit numbers, such as $43 \times 67$, is to use a Place Value Sections model, which shows the relationship the Distributive Property shares with multiplication—the areas of the four smaller rectangles represent the four partial products of the multiplication.

**Expanded Notation**

$\begin{align*}
40 \times 60 &= 2,400 \\
40 \times 7 &= 280 \\
3 \times 60 &= 180 \\
3 \times 7 &= 21 \\
\end{align*}$

$2,881 = 2,400 + 280 + 180 + 21$

**New Groups Above**

$\begin{align*}
\frac{43}{67} \\
\times 40 \\
\underline{+201} \\
\hline
2,881
\end{align*}$

**New Groups Below**

$\begin{align*}
\frac{43}{67} \\
\times 40 \\
\underline{+201} \\
\hline
2,881
\end{align*}$

**Distributive Property** Using expanded notation to find the product of two-digit numbers involves place value because the Distributive Property decomposes the factors into base ten units, and produces the four factor pairs $(40 \times 60, 3 \times 60, 40 \times 7, \text{and} (3 \times 7)$ that represent the partial products of the multiplication.

These lessons demonstrate that although a rectangular area model can be used to represent all multiplication situations, the partial products of those situations can be recorded in different ways.
Multiplication With Decimal Numbers

Decimal Shift Patterns  The activities presented in Lesson 6 enable students to compute decimal products symbolically and explore shift patterns in those products. The first pattern they explore shows that the product of a whole number and a decimal has the same number of decimal places as the decimal factor. Since multiplication is repeated addition, addition is used to verify this relationship.

\[
2 \times 0.3 \text{ ton} = 0.6 \text{ ton} \quad \text{because} \quad 0.3 + 0.3 = 0.6
\]

Students also explore the effect of multiplying by 0.1, 0.01, and 0.001, and again see the pattern that the product of a whole number and a decimal has the same number of decimal places as the decimal factor.

<table>
<thead>
<tr>
<th>x</th>
<th>0.3</th>
<th>0.03</th>
<th>0.003</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(2 \times 0.3)</td>
<td>(2 \times 0.03)</td>
<td>(2 \times 0.003)</td>
</tr>
<tr>
<td></td>
<td>= (2 \times 3 \times 0.1)</td>
<td>= (2 \times 3 \times 0.01)</td>
<td>= (2 \times 3 \times 0.001)</td>
</tr>
<tr>
<td></td>
<td>= 6 \times 0.1</td>
<td>= 6 \times 0.01</td>
<td>= 6 \times 0.001</td>
</tr>
<tr>
<td></td>
<td>= 0.6</td>
<td>= 0.06</td>
<td>= 0.006</td>
</tr>
</tbody>
</table>

The patterns shown in the table above demonstrate how the product of a whole number and a decimal factor can be computed using ones, tens, and hundreds, or using tenths, hundredths, or thousandths. They give students an opportunity to recognize that the shift pattern for multiplying whole numbers is the opposite of the shift pattern for multiplying decimals. In whole number multiplication, the digits in the multiplicand move to the left. When the multiplication consists of one or more decimal factors, the digits shift to the right. Both shifts often require students to use zeros as placeholders.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Decimal Products and Factors  General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. Before students consider decimal multiplication more generally, they can study the effect of multiplying by 0.1 and by 0.01 to explain why the product is ten or a hundred times as small as the multiplicand (moves one or two places to the right). They can then extend their reasoning to multipliers that are single-digit multiples of 0.1 and 0.01 (e.g., 0.2 and 0.02, etc.).
Shift Patterns with Decimals  The activities in Lesson 7 provide students with additional opportunities to explore shift patterns of decimals. Students first work with a real world example of saving a portion of a $213 monthly income.

Leon’s Earnings

\[ $2 1 3 \times 1 = 2 1 3 \]

Students first explore the shift using 0.1 as a factor.

Save 0.1 Each Month

\[ $2 1 3 \times 0.1 = 2 1 . 3 0 \]

Students then explore the shift using 0.01 as a factor.

Save 0.01 Each Month

\[ $2 1 3 \times 0.01 = 2 . 1 3 \]

These activities involve one decimal factor, and lead students to the following generalizations:

- Multiplying by 0.1 produces a shift of one place to the right and produces a product 10 times as small. Multiplying by 0.1 is equivalent to multiplying by \( \frac{1}{10} \) or dividing by 10.
- Multiplying by 0.01 produces a shift of two places to the right and produces a product 100 times as small. Multiplying by 0.01 is equivalent to multiplying by \( \frac{1}{100} \) or dividing by 100.
Two Decimal Factors Also in Lesson 7, students explore shift patterns that occur when both factors are decimals. The symbolic exercises and real world problems they work with lead them to infer the following:

- The number of decimal places in a product is the same as the total number of decimal places in the factors.

Connect Decimals to Whole Numbers and Fractions The algorithms used to multiply decimals are the same as those used for whole numbers. Whether the factors are decimals or whole numbers, the digits in the product are the same. Reminding students of this connection will allow them to focus on the relationships between multiplying whole numbers and multiplying decimals and will promote understanding.

The rules about placing a decimal point in the product connects multiplying decimals to fractions. When multiplying fractions, we multiply the numerators as if they are whole numbers, and then multiply the denominators, for example \( \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} \). The number of zeros in the product of the denominator corresponds to the number of decimal places, which we use to place the decimal point.

Application The activities in Lesson 8 give students an opportunity to practice and apply the decimal shift patterns they learned in Lesson 7. Their work involves computing products symbolically and solving problems in real world contexts.

Properties In Lesson 8, students also complete equations using the Commutative Property of Multiplication, the Associative Property of Multiplication, and the Distributive Property.

Compare and Contrast Shift Patterns In Lesson 9, students compare and contrast shift patterns of whole number multipliers (10 and 100) to those of decimal multipliers (0.1 and 0.01).

Students conclude from these relationships that:

- multiplying by 10 or 0.1 results in a shift of one place.
- multiplying by 100 or 0.01 results in a shift of two places.
- the difference is the direction of the shift.
Powers of 10  Students also explore shift patterns using powers of 10, and infer that the shift pattern of the digits is described by the number of times 10 is used as a factor (i.e. the exponent).

\[
\begin{align*}
10^1 &= 10 \\
10^2 &= 10 \times 10 = 100 \\
10^3 &= 10 \times 10 \times 10 = 1,000 \\
0.4 \times 10 &= 0.4 \times 10^1 = 4 \\
0.4 \times 100 &= 0.4 \times 10 \times 10 = 0.4 \times 10^2 = 40 \\
0.4 \times 1,000 &= 0.4 \times 10 \times 10 \times 10 = 0.4 \times 10^3 = 400
\end{align*}
\]

Generalization  In Lesson 9, students also extend the Big Idea from Lesson 7—the number of decimal places in a product is the same as the total number of decimal places in the factors—to include 5-pattern products, or products such as 0.8 \times 0.5. Although such products are typically written using only one decimal place (0.8 \times 0.5 = 0.4), they extend the Big Idea because there are two decimal places in the factors and two in the product (0.8 \times 0.5 = 0.40) before the product is written in simplest form.

Reasonable Answers

Check Products  The activities and exercises in Lesson 10 remind students that they know a variety of strategies that they can use for checking exact products for reasonableness. One strategy is to use rounding to create an estimate that is compared to an exact answer, and is implemented by students on computations such as these.

<table>
<thead>
<tr>
<th>Estimated Answer</th>
<th>Exact Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>23. 24 \times 39 \approx 20 \times 40 \approx 800</td>
<td>24 \times 39 = 936</td>
</tr>
<tr>
<td>26. 12.3 \times 3.7 \approx 12 \times 4 \approx 48</td>
<td>12.3 \times 3.7 = 45.51</td>
</tr>
</tbody>
</table>

When making estimates, students are encouraged to use patterns and, whenever possible, mental math. For example, a pattern of basic facts and zeros to create an estimate for Exercise 23 above could be 2 \times 4 = 8 and 2 \times 40 = 80. So, 20 \times 40 = 800. The basic fact 2 \times 4 = 8, and the generalization the number of zeros in the factors is equal to the number of zeros in the product, enables many students to complete the estimate using only mental math.
Check for Reasonableness  Also in Lesson 10, students use estimation to check the reasonableness of given products, such as:

\[
\begin{align*}
24.5 \times 4 &= 98 \\
0.56 \times 30 &= 1.68 \\
15.2 \times 2.03 &= 30.856
\end{align*}
\]

Although strategies for checking these products may vary, many students will use rounding to decide. For example:

▶ 24.5 \times 4 = 98 is reasonable because 24.5 is about 25, and 25 \times 4 = 100.

▶ 0.56 \times 30 = 1.68 is not reasonable because 0.56 is about \(\frac{1}{2}\), and \(\frac{1}{2}\) of 30 is 15.

▶ 15.2 \times 2.03 = 30.856 is reasonable because 15.2 is close to 15, 2.03 is close to 2, and 15 \times 2 = 30.

Practice  The activities in Lesson 11 involve practice, giving students an opportunity to apply the concepts they have learned in Unit 4. The activities range from performing symbolic multiplication computations to solving real world multiplication word problems, and involve either one or two decimal factors.

Focus on Mathematical Practices  Lesson 12  

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about multiplying whole numbers and decimals to complete computations related to measurements of insects.