From Our Curriculum Research Project:
Understanding Problem Situations

High-level goals are achieved by enabling all students to enter the mathematical activity at their own level. Teachers accomplish this by using rich and varied language about a given problem so that all students come to understand the problem situation, by mathematizing (focusing on the mathematical features of) a situation to which all students can relate (and that may be generated by a student), and by having students draw models of the problem situation. Cumulative experiencing and practicing of important knowledge skills helps students move through developmental trajectories to more advanced methods. Peer helping provides targeted assistance when necessary. The knowledge of the helper also increases. Assessment provides feedback to all and permits realistic adjustments of proximal learning goals by students and by the teacher.

From Current Research:
Accessible Methods for Multidigit Addition

Method A [New Groups Above, see figure on next page] is an addition algorithm currently appearing in many U.S. textbooks. [. . .] Method B [New Groups Below] is taught in China and has been invented by students in the United States. In this method the new 1 or regrouped 10 (or new hundred) is recorded on the line separating the problem from the answer. [. . .]

Methods A and B both require that children understand what to do when they get 10 or more in a given column. [. . .]

Method C [Show Subtotals], reflecting more closely many students’ invented procedures, reduces the problem [of carrying] by writing the total for each kind of unit on a new line. The carrying-regrouping-trading is done as part of the adding of each kind of unit. Also, Method C can be done in either direction.
Math Drawings Support Thinking and Math Talk  A central result of our research project was the power of having students make math drawings and relate them to mathematical notation. Math drawings are simplified drawings that show quantities and relationships in simple ways. The Visual Math-Talk Community then is an on-going process of building and relating these math drawing visuals and language. Students can begin working from their strength (visual or language), and the continuing connections enable them to develop the other aspect.


Other Useful References: Addition and Subtraction


Getting Ready to Teach Unit 2

Using the Common Core Standards for Mathematical Practice

The Common Core State Standards for Mathematical Content indicate what concepts, skills, and problem solving students should learn. The Common Core State Standards for Mathematical Practice indicate how students should demonstrate understanding. These Mathematical Practices are embedded directly into the Student and Teacher Editions for each unit in Math Expressions. As you use the teaching suggestions, you will automatically implement a teaching style that encourages students to demonstrate a thorough understanding of concepts, skills, and problems. In this program, Math Talk suggestions are a vehicle used to encourage discussion that supports all eight Mathematical Practices. See examples in Mathematical Practice 6.

**Mathematical Practice 1**

**Make sense of problems and persevere in solving them.**

Students analyze and make conjectures about how to solve a problem. They plan, monitor, and check their solutions. They determine if their answers are reasonable and can justify their reasoning.

**TEACHER EDITION: Examples from Unit 2**

**MP.1 Make Sense of Problems**

- **Analyze Relationships**  After students complete Exercises 14–18, ask them to discuss how the metric system relates to our system of place value. In each system, the value of the units are $\times 10$ as you move left and $\div 10$ as you move right.

**MP.1 Make Sense of Problems**

- **Reasonable Answers**  Present students with the following word problem: Mrs. Teal drove 793 miles in October and 542 miles in November. How many total miles did she drive in those two months?

  On the board, write the addition problem 793 + 542 and discuss how rounding to the nearest hundred can be used to make a reasonable estimate of the sum. A reasonable estimate is 1,300 because 793 rounds to 800 and 542 rounds to 500 and $800 + 500 = 1,300$.

---

**Mathematical Practice 1** is integrated into Unit 2 in the following ways:

- Analyze Relationships
- Make a Graph
- Reasonable Answers
Mathematical Practice 2

Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. They can connect diagrams and equations for a given situation. Quantitative reasoning entails attending to the meaning of quantities. In this unit, this involves understanding and generating equivalent decimals and comparing, adding, and subtracting decimals and whole numbers.

**TEACHER EDITION: Examples from Unit 2**

**MP.2 Reason Abstractly and Quantitatively**

Connect Symbols and Words

Ask four more volunteers to write the values of each nonzero digit, 3, 5, 2, and 6 on the board. Place “+” between each term to form an expression.

35,026

30,000 + 5,000 + 20 + 6

Indicate to students that this is the expanded form of the number. Ask the class to add the four numbers together and report the sum. Students should indicate that the sum is the original number in standard form. Guide the class to recognize that the expanded form looks like Secret Code Cards of the number before they are overlapped.

**MP.2 Reason Abstractly and Quantitatively**

Connect Symbols and Models

On the board, draw a vertical number line by tenths from 3.0 to 4.0.

Plot a point at 3.2 on the number line. Discuss how the number line can be used to round 3.2 to the nearest whole number.

- On this number line, in which direction are the numbers increasing? up
- On this number line, in which direction are the numbers decreasing? down
- Is 3.2 closer to 3 or to 4? 3
- What is 3.2 rounded to the nearest whole number? 3

Have a student volunteer plot the point 3.7. Ask what whole number 3.7 would round to. Have students explain how they got their answer. 3.7 is closer to 4 than to 3, or 3.7 is above 3.5 on the number line so it will round to 4.

Mathematical Practice 2 is integrated into Unit 2 in the following ways:

Connect Symbols and Models  |  Connect Symbols and Words
Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Students use stated assumptions, definitions, and previously established results in constructing arguments. They are able to analyze situations and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Students are also able to distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Students can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Talk is a conversation tool by which students formulate ideas and analyze responses and engage in discourse. See also MP.6 Attend to Precision.

What’s the Error? WHOLE CLASS

MP.3, MP.6 Construct Viable Arguments/Critique Reasoning of Others Puzzled Penguin Discuss the Puzzled Penguin activity. Students should see that Puzzled Penguin has added unlike amounts (dollars and cents) and arrived at the wrong total. Have a volunteer write the problem vertically on the board and solve it.

$15.00
+ 0.15
$15.15

Discuss why the problem is set up this way. Encourage students to justify the alignment by talking about the place values. Elicit from them that only like amounts can be added (you can’t add dollars to dimes or pennies).

Mathematical Practice 3 is integrated into Unit 2 in the following ways:

- Compare Methods
- Compare Strategies
- Compare Representations
- Justify Conclusions
- Puzzled Penguin
Mathematical Practice 4
Model with mathematics.

Students can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an equation to solve a problem. Students might draw diagrams to lead them to a solution for a problem. Students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation. They are able to identify important quantities in a practical situation and represent their relationships using such tools as diagrams, tables, graphs, and formulas.

**TEACHER EDITION: Examples from Unit 2**

**MP.4 Model with Mathematics Dimes and Pennies** Ask students how they would use play money to model the problem:

I have 60 cents in one pocket and 20 cents in the other. How much money do I have? $0.80

Have a volunteer write the problem and its answer on the board in a vertical format.

**MP.1, MP.4 Make Sense of Problems/Model with Mathematics Make a Graph** Let each student work independently to construct the bar graph of the sample masses.

**Compare and Order** In Exercise 12, as students write the sample masses in order, they will compare each pair of numbers from the left most place value to the thousandths place as necessary.

Mathematical Practice 4 is integrated into Unit 2 in the following ways:

Dimes and Pennies
Make a Graph
Mathematical Practice 5
Use appropriate tools strategically.

Students consider the available tools and models when solving mathematical problems. Students make sound decisions about when each of these tools might be helpful. These tools might include paper and pencil, a straightedge, a ruler, or the MathBoard. They recognize both the insight to be gained from using the tool and the tool's limitations. When making mathematical models, they are able to identify quantities in a practical situation and represent relationships using modeling tools such as diagrams, grid paper, tables, graphs, and equations.

Modeling numbers in problems and in computations is a central focus in *Math Expressions* lessons. Students learn and develop models to solve numerical problems and to model problem situations. Students continually use both kinds of modeling throughout the program.

**TEACHER EDITION: Examples from Unit 2**

**MP.5 Use Appropriate Tools**
- **MathBoard Modeling**
  Ask a volunteer to demonstrate how to position the MathBoard so the long bar divided into 100 equal parts is at the top. Tell the class that the bar shows one whole. Then ask them to label one end 0 and the other end 1.0, and then label each heavy tick mark by tenths.

**MP.5 Use Appropriate Tools**
- **Model with an Equation**
  Give students an opportunity to write the four equations related to the break apart drawings.

- \[164 + 3,836 = 4,000\]
- \[3,836 + 164 = 4,000\]
- \[4,000 - 3,836 = 164\]
- \[4,000 - 164 = 3,836\]
- \[3.57 + 4.67 = 8.24\]
- \[4.67 + 3.57 = 8.24\]
- \[8.24 - 4.67 = 3.57\]
- \[8.24 - 3.57 = 4.67\]

Struggling students may benefit from break apart drawings as they relate addition and subtraction or solve word problems with unknown addends.

**Mathematical Practice 5** is integrated into Unit 2 in the following ways:
- Decimal Secret Code Cards
- MathBoard Modeling
- Meter Stick
- Model the Mathematics
- Model with an Equation
- Secret Code Cards
**Mathematical Practice 6**

**Attend to precision.**

Students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. They are careful about specifying units of measure to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. Students give carefully formulated explanations to each other.

**MATH TALK**

Help Small Groups of students answer Questions 14–15 on page 48 of the Student Book. Discuss why these word problems exemplify these properties.

Then assign each group to describe a situation that illustrates the Commutative Property.

**MATH TALK in ACTION**

Look at the vertical scale in Puzzled Penguin’s graph. What do you notice about it? How can it be fixed?

**Mata:** Puzzled Penguin uses a different scale between 0 and 1.0, 1.1 and 1.3, and 1.3 and 1.4. The intervals should be the same.

**Charles:** I think Puzzled Penguin should start at 0 and use intervals of 0.1 meter.

**Arthur:** The range of the data is from 1.0 m to 1.5 m. That would make the graph pretty tall.

**Eva:** There is a way to show that we are skipping part of the scale by having a zigzag in the line of the scale between 0 and the first number. Then we could make each interval 0.1 meters and not have to make the graph so tall.

**Mathematical Practice 6** is integrated into Unit 2 in the following ways:

- **Describe a Method**
- **Explain a Representation**
- **Explain a Solution**
- **Puzzled Penguin**
- **Verify a Solution**
Mathematical Practice 7
Look for and make use of structure.

Students analyze problems to discern a pattern or structure. They draw conclusions about the structure of the relationships they have identified.

**TEACHER EDITION: Examples from Unit 2**

**MP.7 Look for Structure** Writing the zero before the decimal point helps make decimal numbers look more like their equivalent fractions. For example, 0.1 and \( \frac{1}{10} \), 0.01 and \( \frac{1}{100} \), and 0.001 and \( \frac{1}{1,000} \).

**Lesson 1 ACTIVITY 1**

**MP.7 Look for Structure** Look for a Pattern As they work to complete Exercises 1–7, ask students to discuss the patterns they see in the Place Value Chart on Student Book page 41 (especially those relating metric units to money and to place value relationships).

Lesson 4 ACTIVITY 2

**Mathematical Practice 7** is integrated into Unit 2 in the following ways:

- Identify Relationships
- Look for a Pattern
Math and Our Solar System

To describe distances on Earth, you do not need to use units of measure greater than thousands of miles. In space, however, distances are vast, and greater units of measure are used to describe those distances. Distances between objects in our solar system usually involve many millions of miles.

Scientists express the distances in astronomical units (AU). One AU is the distance from the Earth to our Sun, which is about 93 million miles.

1 AU \approx 93,000,000 \text{ miles}

When you work with distances in our solar system, it is easier to add and subtract astronomical units than it is to add and subtract numbers in the millions.

Solve.

1. Venus orbits 0.72 AU from the Sun. Mercury's orbit is 0.33 AU closer. Explain why subtraction is used to find the distance of Mercury's orbit from the Sun. Then find the distance.

\begin{align*}
\text{Venus orbit:} & \quad 0.72 \text{ AU} \\
\text{Mercury orbit:} & \quad 0.72 - 0.33 = 0.39 \text{ AU}
\end{align*}

Show your work.

The distances are being compared, and subtraction is used to compare; 0.39 AU.

The table below shows the distances in astronomical units (AU) of the planets from our Sun. Since the planets Mercury and Venus are closer to the Sun than Earth, their distance from the Sun is less than 1 AU. Outer planets such as Jupiter and Neptune have distances greater than 1 AU.

Solve. Compute the distance between the planets' orbits.

2. Venus and Earth

3. Earth and Mars

4. Jupiter and Saturn

5. Mercury and Neptune

6. Write Mercury's orbital distance (in AU) from the Sun as a fraction.

7. The sum of the orbital distances from the Sun of which four planets is closest to the orbital distance from the Sun to Neptune?

The table below shows the distances in astronomical units (AU) of the planets from our Sun. Since the planets Mercury and Venus are closer to the Sun than Earth, their distance from the Sun is less than 1 AU. Outer planets such as Jupiter and Neptune have distances greater than 1 AU.

Solve. Compute the distance between the planets' orbits.

\begin{align*}
\text{Venus orbit:} & \quad 0.72 \text{ AU} \\
\text{Mercury orbit:} & \quad 0.72 - 0.33 = 0.39 \text{ AU}
\end{align*}

Show your work.

Remind students that the same properties can be used to simplify the fraction addition.
Getting Ready to Teach Unit 2

Learning Path in the Common Core Standards

In this unit, students extend their understanding of the base-ten system to decimals to the thousandths place, building on their Grade 4 work with tenths and hundredths.

Concrete materials, number lines, and visual drawings are used throughout the unit to illustrate important number and operation concepts. Students observe that the process of composing and decomposing a base-ten unit is the same for decimals as for whole numbers and the same methods of recording numerical work can be used with decimals as with whole numbers.

Visual models and real world situations are used throughout the unit to illustrate important decimal concepts.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to the students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin.

- Lesson 3: Incorrectly writing zeros to make equivalent decimals.
- Lesson 5: Not aligning addends by place value when finding a sum
- Lesson 6: Subtracting greater digits from lesser digits without ungrouping
- Lesson 9: Graphing data using inconsistent axes intervals

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.
Decimals as Equal Divisions

Relating Fractions and Decimals  Visual models are used to show fraction and decimal notations, which are different ways to represent a whole divided into equal parts.

In fraction notation, the denominator represents the number of equal parts a whole is divided into, and the numerator represents the number of parts being described.

\[ \frac{1}{4} \text{ of 4 equal parts} \]

Decimal notation shows the number of places to the right of the ones place. The tenths place represents the division of 1 whole into 10 equal parts.

\[ \frac{1}{10} \text{ of 10 equal parts} \]

The hundredths place represents the division of 1 whole into 100 equal parts, and shows each tenth divided into 10 equal parts.

\[ \frac{1}{100} \text{ of 100 equal parts} \]

The thousandths place represents the division of 1 whole into 1,000 equal parts, and shows each hundredth divided into 10 equal parts.

\[ \frac{1}{1,000} \text{ of a penny} \]

Students read decimals as if they are fractions, and use repeated reasoning to recognize patterns such as when a denominator is a power of 10, the number of zeros in the denominator is the number of places to the right of the ones place in an equivalent decimal.
**Understanding Place Value** A place value chart enables students to see how adjacent place values are related. Place value relationships are used when comparing two or more numbers.

A place value chart also helps students understand ten-for-one ungroupings and one-for-ten groupings when finding sums and differences of whole numbers and decimals. The bottom row of the table shows coins and bills that represent base-ten place value.

**Secret Code Cards** The use of Secret Code Cards provides a way for students to see the expanded form of numbers.

**Modeling Whole Numbers** To model a whole number such as 2,435 students select cards representing 2 thousands, 4 hundreds, 3 tens, and 5 ones. When spread out and arranged in descending order from left to right, the cards show the expanded form of the number.

Overlapping the cards reveals the standard form of the number.

The numbers in the upper left corners of the cards represent the number of thousands, hundreds, tens, and ones that make up the whole number.
Modeling Decimal Numbers  To model a decimal such as 5.34 students select cards representing 5 ones, 3 tenths, and 4 hundredths. When spread apart and arranged in descending order from left to right, the cards show the expanded form of the number.

\[
\begin{align*}
5 & \quad 0.3 & \quad 0.04 \\
5 + 0.3 + 0.04 & \\
& 5.34
\end{align*}
\]

Overlapping the cards reveals the standard form of the number.

\[
\begin{array}{cccc}
5 & 0.3 & 0.04 \\
& & & \\
& & & \\
& & & \\
& & & \\
\end{array}
\]

The numbers in the upper corners of the cards represent the number of ones, tenths, and hundredths that make up the decimal.

Secret Code Cards help students read whole numbers and decimals and write them in expanded, standard, and word form.

- Expanded Form: \(5 + 0.3 + 0.04\)
- Standard Form: 5.34
- Word Form: five and thirty-four hundredths

*from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN*

Children can use layered place value cards to see the 10 “hiding” inside any teen number. Such decompositions can be connected to numbers represented with objects and math drawings.
Equate and Compare Thousandths

Comparing Decimals  The tenths and hundredths bars and grids in Lesson 3 represent different ways to model decimals, and provide a way for students to see how two decimal quantities compare.

Tenths and Hundredths Bars  A bar is used to represent 1 whole, and incorporates tenths and hundredths divisions to represent a part of that whole. The shading below, for example, represents 0.62, and enables students to see the expanded form of 0.62 as 0.6 + 0.02.

Shading also provides a visual way for students to compare two decimal values. By shading a bar to represent each value, students can see that the bar with a greater number of tenths or hundredths shaded represents the greater value. Comparing tenths and hundredths in this visual way forms the foundation of comparing in a more symbolic way when students compare the digits in the tenths place (i.e., the number of tenths), followed if necessary by comparing the digits in the hundredths place (i.e. the number of hundredths).

Area Grids  Lesson 3 includes other visual models to enhance understanding of place value. Ten-by-ten grids are used to represent 1 whole. Since the area of each grid is 10 × 10 or 100 unit squares, students shade any number of rows or columns to represent tenths, and any number of unit squares to represent hundredths. To compare 0.4 and 0.36 for example, students shade grids as shown below.

The shading enables students to form conclusions such as these:

- Since more of the 0.4 grid is shaded, 0.4 > 0.36.
- Since more tenths of the 0.4 grid are shaded, 0.4 > 0.36.
- Since more hundredths of the 0.4 grid are shaded, 0.4 > 0.36.
**Other Ways to Compare** Students also use number lines, Secret Code Cards, and money to enhance their understanding of place value and how decimals compare.

A number line is a model of distance from zero, and is a visual way for students to compare two decimals. Students conclude from their work with number lines that any value on a number line is greater than any value to its left because it consists of more tenths, more hundredths, or more tenths and hundredths.

![Number Line Diagram]

Secret Code Cards for a decimal such as 0.497 help students see that $0.4 > 0.09$ and $0.09 > 0.007$.

![Secret Code Cards Diagram]

Students also use coins that represent our base-ten number system (i.e., dimes and pennies) to compare decimals. Since 10 dimes = 1 dollar and 100 pennies = 1 dollar, dimes represent tenths of a dollar and pennies represent hundredths. Modeling with coins leads students to generalize that a greater amount has a greater number of dimes, and when the number of dimes is the same, the greater amount has the greater number of pennies.

Students use a variety of models in Lesson 3 to build and enhance their understanding of place value. Different models appeal to different learning styles and give all students an opportunity to understand the concepts.
Add and Subtract Whole Numbers and Decimals

Decimal and Whole Number Algorithms  Lessons 4–6 involve using addition and subtraction to find decimal sums and differences. The decimals used for these computations are both greater than and less than 1.

Place Value  In Lesson 4, the place value chart makes its second appearance. The chart again shows the relationships that adjacent places in our base-ten numeration system share. It also incorporates metric units because the metric system of measure is also a base-ten system. For any two adjacent places or units in either system, the value of the place to the left is 10 times the value of the place to its right, and the value of a place to the right is \( \frac{1}{10} \) the value of the place to its left. Both systems involve powers of 10, and the metric conversions that students are asked to complete in this lesson reinforce their understanding of place value.

Whole Number Addition Strategies  Prior to finding a variety of decimal sums and differences in Lesson 5, students share strategies for finding the sum of two whole numbers. They conclude from the discussion that strategies used to find whole number sums can also be used to find decimal sums. Students then discuss and demonstrate those strategies using 769 and 584 as addends.

The New Groups Below strategy involves recording groupings under the addends.

\[
\begin{array}{c}
\text{769} \\
+ \text{584} \\
\hline \\
1,353
\end{array}
\]

The New Groups Above strategy involves recording groupings above the addends.

\[
\begin{array}{c}
\hline \\
\text{1,353} \\
\text{769} + \text{584} \\
\hline
\end{array}
\]

The Subtotal strategy involves adding hundreds, tens, and ones separately.

Left to Right:  
\[
\begin{array}{c}
1200 \\
140 \\
\hline \\
13 + 13 \\
\hline \\
1,353
\end{array}
\]

Right to Left:  
\[
\begin{array}{c}
13 \\
140 \\
\hline \\
\text{1,353} + 1200 \\
\hline
\end{array}
\]

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Add and Subtract Decimals  
Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals as they used for adding and subtracting whole numbers.
Decimal Addition Strategies  Similar strategies can be used to find the sum of decimal addends.

The New Groups Below strategy involves recording groupings under the addends.

\[
\begin{array}{c}
44.25 \\
+ \ 8.96 \\
\hline \\
53.21 \\
\end{array}
\]

The New Groups Above strategy involves recording groupings above the addends.

\[
\begin{array}{c}
44.25 \\
+ \ 8.96 \\
\hline \\
53.21 \\
\end{array}
\]

Connect Decimals to Whole Numbers and Fractions  To successfully find the sum of decimal addends, students must understand the importance of adding like units. A simple way for students to make sure they add like units is to make sure the decimal points in the addends are aligned vertically. Aligning decimal points to add decimals is comparable to finding common denominators when adding fractions.

Students' success with decimal sums also depends on their understanding of ungrouping. Students have already spent a great deal of time ungrouping 10 hundreds as 1 thousand, 10 tens as 1 hundred, and 10 ones as 1 ten. When finding decimal sums, they learn that 10 tenths are grouped as 1 one and 10 hundredths are grouped as 1 tenth.

When the number of decimal places in two or more addends is different, some students may find it helpful to write equivalent addends. For example, to add a whole number and a decimal, a decimal point and any number of zeros can be written to the right of the whole number.

\[
1 = 1.0 = 1.00 = 1.000
\]

To add two decimals that have a different number of decimal places, any number of zeros can be written to the right of the decimal.

\[
0.1 = 0.10 = 0.100
\]

Writing equivalent addends not only makes it easier for some students to ensure they add only like units, it also may reduce the likelihood of computation errors.
Connect Decimals to Whole Number and Fractions  In Lesson 6, students learn that our base-ten numeration system has a uniform structure which enables students to use the same place value understandings to subtract whole numbers and decimals as they used to add whole numbers and decimals. In other words, many of the same strategies can be used to add and subtract. For example, the New Groups Above strategy described earlier can be used.

\[
\begin{array}{c}
\phantom{0}13.86 \\
\text{-} 4.91 \\
\hline
8.95
\end{array}
\]

As with decimal addition, students must understand the importance of subtracting like units when subtracting decimals (e.g., ones from ones, tenths from tenths), and this is accomplished by making sure the decimal points are aligned before performing the computation.

Students’ success finding decimal differences also depends on their understanding of ungrouping. Students have already spent a great deal of time ungrouping 1 thousand as 10 hundreds, 1 hundred as 10 tens, and 1 ten as 10 ones. When finding decimal differences, they must understand that 1 tenth is ungrouped as 10 hundredths, and 1 hundredth is ungrouped as 10 thousandths.

\[
0.1 = 0.10 \text{ and } 0.01 = 0.010
\]
Properties and Strategies

Lesson 7

Students learn that addition properties have many applications.

Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative Property of Addition</td>
<td>( a + b = b + a )</td>
</tr>
<tr>
<td>Associative Property of Addition</td>
<td>( (a + b) + c = a + (b + c) )</td>
</tr>
<tr>
<td>Distributive Property</td>
<td>( a \cdot (b + c) = (a \cdot b) + (a \cdot c) )</td>
</tr>
</tbody>
</table>

Mathematics often involves computations, and addition properties can be used to make computations easier to perform. For example, simplifying the expression below from left to right requires grouping 14 tens as 1 hundred and 4 tens.

\[
91 + 50 + 50 = 141 + 50 = 191
\]

The Associative Property of Addition enables students to add 50 + 50 first, and provides them with an opportunity to perform the entire computation using only mental math.

\[
91 + 50 + 50 = 91 + 100 = 191
\]

The Distributive Property can be used to simplify many computations that involve multiplication and addition.

\[
(800 \cdot 9) + (200 \cdot 9) = (800 + 200) \cdot 9 = 1,000 \cdot 9 = 9,000
\]

Generalize Properties of addition can also be used by students in a more general way. In the computation below, for example, students who group 20,000 with 80,000 and 30,000 with 70,000 can perform the entire computation \((100,000 + 100,000 + 49,000 = 249,000)\) using only mental math.

\[
\begin{align*}
30,000 \\
20,000 \\
80,000 \\
49,000 \\
\text{+ 70,000} \\
\hline
249,000
\end{align*}
\]

Real World Applications Students also have opportunities in Lesson 7 to use these properties in real world situations.
Round and Estimate
With Decimals

Rounding When finding sums and differences, students are not always asked to give exact answers. There are situations in which an estimate (or approximation) of an answer is sufficient.

In Lesson 8, students practice rounding whole numbers and decimals to the nearest one, tenth, and hundredth, as well as infer the place to which a variety of numbers have been rounded.

A number line is one method students can use to round numbers. For example, to round 4.7 to the nearest whole number, students must make decisions such as:

- 4.7 is between the whole numbers 4 and 5.
- 4.5 is halfway between 4 and 5.
- 4.7 is to the right of 4.5.
- 4.7 rounds to 5 because it is closer to 5 than to 4.

Using a number line to round helps students make sense of symbolic rounding, which involves comparing the digit in the place to the right of the rounding place to 5—if the digit is less than 5, the digit in the rounding place does not change, and if the digit is 5 or more, the digit in the rounding place increases by 1.

Estimating Students should understand the importance of checking computations for reasonableness. One way to check is to apply a property (whenever possible) and perform the computation a second time. However, such a method can involve a great deal of time and be discouraging. A more attractive alternative for students is to estimate an answer. In other words, use their understanding of place value to get a sense of what to expect for an answer. For example, if two decimals are greater than 1 and less than 2, students should expect the sum to be greater than $1 + 1$ or 2 and less than $2 + 2$ or 4.

Regardless of the method students use to decide reasonableness, all of the methods involve number sense. The more students practice developing number sense, the stronger their understanding of our number system becomes.
Graph With Decimal Numbers

In Lesson 9 students examine a decimal bar graph and compare and contrast it to the characteristics of a whole-number bar graph. Students then use the discussion as a springboard to make a bar graph that displays a given set of decimal data.

Focus on Mathematical Practices

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about finding decimal sums and differences to compute distances in our solar system.