Putting Research into Practice

From Our Curriculum Research Project: Math Talk Is Important

A significant part of the collaborative classroom culture in *Math Expressions* is the frequent exchange of problem-solving strategies, or Math Talk. The benefits of Math Talk are multiple. Describing one’s methods to another person can clarify one’s own thinking as well as clarify the matter for others. Another person’s approach can supply a new perspective, and frequent exposure to different approaches tends to engender flexible thinking. Math Talk creates opportunities to understand errors and permits teachers to assess students’ understanding on an ongoing basis. It encourages students to develop their language skills, both in math and in everyday English. Finally, Math Talk enables students to become active helpers and questioners, creating student-to-student talk that stimulates engagement and community.

From Current Research: Models of Fractions

During Grades 3–5, students should build their understanding of fractions as parts of a whole and as division. They will need to see and explore a variety of models of fractions, focusing primarily on familiar fractions such as halves, thirds, fourths, fifths, sixths, eighths, and tenths.

By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions.

Students should develop strategies for ordering and comparing fractions, often using benchmarks such as $\frac{1}{2}$ and 1. For example, fifth graders can compare fractions such as $\frac{2}{5}$ and $\frac{5}{8}$ by comparing each with $\frac{1}{2}$; one is a little less than $\frac{1}{2}$ and the other is a little more.
By using parallel number lines, each showing a unit fraction and its multiples (see Fig. 5.1), students can see fractions as numbers, note their relationship to 1, and see relationships among fractions, including equivalence. They should also begin to understand that between any two fractions, there is always another fraction.

![Parallel number lines with unit fractions and their multiples](image-url)

Fig. 5.1. Parallel number lines with unit fractions and their multiples


Other Useful References: Fractions


Getting Ready to Teach Unit 1

Using the Common Core Standards for Mathematical Practice

The Common Core State Standards for Mathematical Content indicate what concepts, skills, and problem solving students should learn. The Common Core State Standards for Mathematical Practice indicate how students should demonstrate understanding. These Mathematical Practices are embedded directly into the Student and Teacher Editions for each unit in Math Expressions. As you use the teaching suggestions, you will automatically implement a teaching style that encourages students to demonstrate a thorough understanding of concepts, skills, and problems. In this program, Math Talk suggestions are a vehicle used to encourage discussion that supports all eight Mathematical Practices. See examples in Mathematical Practice 6.

COMMON CORE Mathematical Practice 1
Make sense of problems and persevere in solving them.

Students analyze and make conjectures about how to solve a problem. They plan, monitor, and check their solutions. They determine if their answers are reasonable and can justify their reasoning.

TEACHER EDITION: Examples from Unit 1

MP.1 Make Sense of Problems
Approach Problem 19 strategically so students don’t add together every possible pair of numbers to see if the sum is 10 tons or less. Break the class into Small Groups and ask them to brainstorm for strategies to make solving the problem easier. Have each group share its ideas with the class.

Lesson 9 ACTIVITY 2

MP.1 Make Sense of Problems Check Answers Discuss how we can check that 12 11/12 cups is a reasonable answer. Here is a possible method.

Round to the Nearest Whole Number
Round 23/4 up to 3 and round 4 1/3 down to 4: 20 – (3 + 4) = 13 (or equivalently, 20 – 3 – 4 = 13). The answer should be close to 13 cups. So, 12 11/12 cups is a reasonable answer.

Lesson 12 ACTIVITY 1

Mathematical Practice 1 is integrated into Unit 1 in the following ways:

Analyze the Problem
Check Answers
MathBoard Modeling
Represent the Problem
Mathematical Practice 2
Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. They can connect diagrams and equations for a given situation. Quantitative reasoning entails attending to the meaning of quantities. In this unit, this involves understanding and generating equivalent fractions and comparing, adding, and subtracting fractions and mixed numbers.

TEACHER EDITION: Examples from Unit 1

MP.2 Reason Abstractly and Quantitatively
Connect Symbols and Words

It may help students if they write the unit fractions as word names.

\[
\begin{align*}
\text{one third} + \text{one third} + \text{one third} &= \text{three thirds, or 1} \\
\text{one third} + \text{one third} + \text{one third} &= \text{three thirds, or 1} \\
\text{one third} + \text{one third} &= \text{two thirds} \\
\text{eight thirds} &= \text{two and two thirds}
\end{align*}
\]

Lesson 5 ACTIVITY 1

Mathematical Practice 2 is integrated into Unit 1 in the following ways:

Connect Symbols and Models
Connect Symbols and Words

MP.2 Reason Abstractly and Quantitatively
Connect Symbols and Models

Have students look at the fraction bar at the top of Student Book page 17. Let students lead the discussion as much as possible.

- The first bar shows \( \frac{1}{2} + \frac{1}{3} \). Does it tell us what the total is called? No.
- How can we decide what to call the total? Divide the bar, using a fraction that can rename halves and thirds, a unit fraction that can make \( \frac{1}{2} \) and also \( \frac{1}{3} \).
- How can we find a fraction that does that? Multiply the two denominators: \( 2 \times 3 \).

Lesson 7 ACTIVITY 2
COMMON CORE

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Students use stated assumptions, definitions, and previously established results in constructing arguments. They are able to analyze situations and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Students are also able to distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Students can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Talk is a conversation tool by which students formulate ideas and analyze responses and engage in discourse. See also MP.6 Attend to Precision.

TEACHER EDITION: Examples from Unit 1

MP.3, MP.6 Construct Viable Arguments/Critique Reasoning of Others Puzzled Penguin Give students a minute or two to read the Puzzled Penguin letter and think about how they would respond. Choose volunteers to share their responses and discuss them as a class. The letter emphasizes the need to multiply both parts of a fraction by the same number in order to find an equivalent fraction.

Mathematical Practice 3 is integrated into Unit 1 in the following ways:

- Compare Methods
- Compare Strategies
- Puzzled Penguin

MP.3 Construct Viable Arguments Compare Strategies For each problem, choose a pair to present the solution. Then ask if anyone solved the problem a different way. If so, ask them to present the strategy they used.

Lesson 12 ACTIVITY 1
**Mathematical Practice 4**

Model with mathematics.

Students can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an equation to solve a problem. Students might draw diagrams to lead them to a solution for a problem.

Students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation. They are able to identify important quantities in a practical situation and represent their relationships using such tools as diagrams, tables, graphs, and formulas.

**MP.4 Model with Mathematics**

Write an Equation

Ask if anyone wrote an equation to represent their calculations, and discuss the two equations below. In both, \( f \) represents the amount of flour left in the bag. Be sure students understand that, in the first equation, the parentheses indicate that we do the addition first and then we subtract the result from 20.

\[
f = 20 - (2 \frac{3}{4} + 4 \frac{1}{3}) \quad f = 20 - 2 \frac{3}{4} - 4 \frac{1}{3}
\]

**Mathematical Practice 4** is integrated into Unit 1 in the following ways:

Describe a Method
MathBoard Modeling
Represent the Problem
Write an Equation
**COMMON CORE**

**Mathematical Practice 5**

*Use appropriate tools strategically.*

Students consider the available tools and models when solving mathematical problems. Students make sound decisions about when each of these tools might be helpful. These tools might include paper and pencil, a straightedge, a ruler, or the MathBoard. They recognize both the insight to be gained from using the tool and the tool’s limitations. When making mathematical models, they are able to identify quantities in a practical situation and represent relationships using modeling tools such as diagrams, grid paper, tables, graphs, and equations.

Modeling numbers in problems and in computations is a central focus in *Math Expressions* lessons. Students learn and develop models to solve numerical problems and to model problem situations. Students continually use both kinds of modeling throughout the program.

**TEACHER EDITION: Examples from Unit 1**

**MP.5 Use Appropriate Tools** MathBoard Model

Be sure to include the following points in a class discussion about the MathBoard:

- The top bar shows one whole. The other bars show this same whole divided into different numbers of equal parts.
- The bars are labeled *halves*, *thirds*, *fourths*, and so on to indicate how the bars are divided.
- The number of parts increases with each bar, while the size of the parts decreases.

**Unit Fractions** Ask students what fraction each part of the halves bar represents, $\frac{1}{2}$. Students should label the first part with this fraction as shown. Explain that students can think of $\frac{1}{2}$ as 1 of 2 equal parts.

**Mathematical Practice 5** is integrated into Unit 1 in the following ways:

- Draw a Diagram
- MathBoard Modeling
- Model the Mathematics
**Mathematical Practice 6**

**Attend to precision.**

Students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. They are careful about specifying units of measure to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. Students give carefully formulated explanations to each other.

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### TEACHER EDITION: Examples from Unit 1

#### MP.6 Attend to Precision Describe a Method

Write the fraction $\frac{9}{4}$ on the board, and ask students to change it to a mixed number. Send several volunteers to the board while the others work at their seats. Encourage other students to contribute to the explanations or to ask questions.

#### MATH TALK

Choose volunteers to explain how they determined which fraction is greater. Encourage students to use the idea of unit fractions in their explanations. Students should understand that the unit fractions for both fractions are the same size (specifically, both are fifths). Because $\frac{4}{5}$ has 4 unit fractions ($\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$) and $\frac{3}{5}$ has only 3 unit fractions ($\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$), $\frac{4}{5}$ is greater.

#### MP.6 Attend to Precision Explain a Solution

Have Small Groups work on the exercises on Student Book page 23. Then discuss the answers to Exercises 10–12 as a class. Allow students to explain in their own words how they found a common denominator for the fractions in each group. Encourage other students to ask questions if an explanation is unclear to them.

#### MATH TALK

Students describe Puzzled Penguin’s error and explain how to correct it.

**What is the mistake in Puzzled Penguin’s work?**

**Zander:** The ungrouping is not right. Puzzled Penguin should have changed the 5 to a 4.

**Mica:** Yes, to get $\frac{7}{9}$, you have to take $\frac{5}{9}$, which is 1 whole, from the $\frac{5}{9}$. That means you have to change the 5 to a 4.

**Alice:** I would tell Puzzled Penguin to add some more steps. That’s what I do.

**Alice, can you explain what you would do?**

**Alice:** I would write 5 as $4 + \frac{5}{9}$ and then add the $\frac{3}{5}$.

**Will you come to the board and show us?**

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**Mathematical Practice 6** is integrated into Unit 1 in the following ways:

- Describe a Method
- Puzzled Penguin
- Explain a Solution
- Verify Solutions
COMMON CORE
Mathematical Practice 7
Look for and make use of structure.

Students analyze problems to discern a pattern or structure. They draw conclusions about the structure of the relationships they have identified.

TEACHER EDITION: Examples from Unit 1

MP.7 Look for Structure  Look for a Pattern
Introduce the term simplify to produce an equivalent fraction with lesser numerator and denominator. Students should complete Exercises 7–11 on Student Book page 5. Then ask them to cover the whole page to the right of 1_3.

• How many fifteenths are grouped together to make 1_3? 5
• How can you find the number of thirds in 10_15? Divide 10 and 15 by 5.
• How many twelfths are grouped together to make 1_3? 4
• How can you find the number of thirds in 8_12? Divide 8 and 12 by 4.
• How many ninths are grouped together to make 1_3? 3
• How can you find the number of thirds in 6_9? Divide 6 and 9 by 3.

MP.7 Look for Structure  Identify Relationships
During the discussion, call attention to the ways in which the denominators in each pair of fractions are related.

For 3_4 and 5_8,

▷ One denominator is a factor of the other (4 is a factor of 8).
▷ The greater denominator, 8, is the least denominator that will work as the common denominator.
▷ Using 8 as the common denominator requires us to rewrite only one of the fractions.

For 4_5 and 5_7,

▷ 1 is the only number that is a factor of both denominators.
▷ The product of 5 and 7 is the least denominator that will work as the common denominator.

For 7_6 and 5_6,

▷ 3 is a factor of both denominators.
▷ 18, which is less than the product of the denominators, will work as a common denominator.

Mathematical Practice 7 is integrated into Unit 1 in the following ways:

Identify Relationships
Look for a Pattern
Math and Bird Hotels

Have you ever seen a birdhouse? Birdhouses offer birds a place to rest and keep their eggs safe from predators. Purple martins are birds that nest in colonies. Purple martin birdhouses, like the one at the right, are sometimes called bird hotels.

Suppose that a fifth grade class has decided to build a two-story purple martin bird hotel. Each story of the hotel will have six identical compartments. One story is shown below.

The bird hotel will be made from wood, and each story will have the following characteristics.

1. The inside dimensions of each compartment measure $7\frac{1}{2}$ in. by $7\frac{1}{2}$ in. by $7\frac{1}{2}$ in.
2. The wood used for exterior walls is $\frac{5}{8}$-in. thick.
3. The wood used for interior walls is $\frac{1}{4}$-in. thick.

Calculate the length, width, and height of one story. Do not include a floor or ceiling in your calculations.

1. length
2. width
3. height

Purple martins migrate great distances, spending winter in South America and summer in the United States and Canada. A diet of flying insects supplies purple martins with the energy they need to complete such long flights.

Suppose a purple martin migrates about 4,150 miles to South America each fall and about 4,150 miles back to North America each spring. Also suppose the purple martin performs this round trip once each year for three years.

7. At the Equator, the distance around Earth is about 24,900 miles. About how many times around Earth would the purple martin described above fly during its migrations?
Getting Ready to Teach Unit 1

Learning Path in the Common Core Standards

In this unit, students study fractions and mixed numbers. They find equivalent fractions, compare fractions, and add and subtract fractions and mixed numbers.

Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent problems with like denominators. They make reasonable estimates of the sums and differences.

Visual models and real world situations are used throughout the unit to illustrate important fraction concepts.

Help Students Avoid Common Errors

*Math Expressions* gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin.

- **Lesson 3:** Generating an equivalent fraction by multiplying only one part of the fraction
- **Lesson 5:** Interpreting a mixed number as a whole number times a fraction
- **Lesson 6:** When ungrouping a mixed number to subtract, forgetting to reduce the whole number part by 1
- **Lesson 7:** Adding fractions by adding numerators and adding denominators
- **Lesson 9:** Adding mixed numbers by adding numerators and adding denominators; when subtracting mixed numbers, subtracting the lesser fraction from the greater fraction, even though the lesser fraction is in the minuend
- **Lesson 11:** Not checking answers for reasonableness

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.
**Unit Fractions** The fraction bars on the MathBoard are used to discuss unit fractions. A unit fraction has the form \( \frac{1}{n} \), where \( n \) is the number of equal parts the whole is divided into. Students recall that a unit fraction represents one of those parts of a whole. For example, the unit fraction \( \frac{1}{5} \) represents one of five equal parts of a whole.

The MathBoard fraction bars allow students to observe that unit fractions with greater denominators represent smaller parts of a whole. For example, \( \frac{1}{5} \) is smaller than \( \frac{1}{3} \). Students can reason that this makes sense because the more parts the same whole is divided into, the smaller each part must be.

**Non-Unit Fractions** Fraction bars allow students to see how a whole and other non-unit fractions are built by combining (adding) unit fractions.

- Any fraction that is not a unit fraction can be built by adding unit fractions. For example, \( \frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \). This can also be expressed as \( \frac{3}{5} = 3 \times \frac{1}{5} \).

  ![Fifths fraction bar](image)

- The denominator of a fraction tells the number of unit fractions in the whole. For example, for \( \frac{3}{5} \), the whole is made up of five unit fractions (specifically, five fifths, or five \( \frac{1}{5} \)s).

- The numerator of a fraction tells the number of unit fractions in the fraction. For example, the fraction \( \frac{3}{5} \) is made up of three unit fractions—in this case, three fifths, or three \( \frac{1}{5} \)s.

Viewing non-unit fractions as sums of unit fractions helps students avoid common errors in adding and subtracting fractions (including adding numerators and denominators, not just numerators).

\[
\frac{3}{7} + \frac{2}{7} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}
\]
**Equivalent Fractions**

**Fraction Bars** Students begin their work with equivalent fractions by looking at the portion of each fraction bar that is equivalent to $\frac{1}{2}$. They consider what we must do both to the fraction-bar model for $\frac{1}{2}$ and to the numerical fraction $\frac{1}{2}$ to form equivalent fractions. For example, to go from the model for $\frac{1}{2}$ to the model for $\frac{3}{6}$, we must divide each half into three equal parts. (We say we must 3-split $\frac{1}{2}$.) To get from the numerical fraction $\frac{1}{2}$ to $\frac{3}{6}$, we must multiply both the numerator and the denominator by 3.

**Number Lines** On a number line, students see that they can make equivalent fractions by dividing each interval into smaller intervals (that is, by dividing each unit fraction into smaller unit fractions). For example, by dividing each third into two equal intervals to make sixths, we can see that $\frac{\frac{2}{3}}{3}$ is equivalent to $\frac{\frac{4}{6}}{6}$. Dividing each interval in two parts is mathematically equivalent to multiplying both the numerator and denominator of $\frac{2}{3}$ by 2.

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**from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS**

**Equivalent Fractions** Students can use area models and number line diagrams to reason about equivalence. They see that the numerical process of multiplying the numerator and denominator of a fraction by the same number, $n$, corresponds physically to partitioning each unit fraction piece into $n$ smaller equal pieces. The whole is then partitioned into $n$ times as many pieces, and there are $n$ times as many smaller unit fraction pieces as in the original fraction.
Students also make equivalent fractions on the number line by grouping intervals to make larger intervals (in other words, by grouping unit fractions to make greater unit fractions). For example, making groups of 5 fifteenths makes thirds. The fraction \(\frac{10}{15}\) becomes the equivalent fraction \(\frac{2}{3}\). This is equivalent to dividing the numerator and denominator by 5.

\[
\frac{10 \div 5}{15 \div 5} = \frac{2}{3}
\]

In *Math Expressions*, we use *simplify* to refer to the process of creating an equivalent fraction by dividing both parts of a fraction by the same number. We use *unsimplify* to refer to the process of creating an equivalent fraction by multiplying both parts by the same number. For example, we can simplify \(\frac{9}{12}\) by dividing both parts by 3 to get \(\frac{3}{4}\). We can unsimplify \(\frac{2}{5}\) by multiplying both parts by 3 to get \(\frac{6}{15}\).

**Multiplication Table** Lesson 3 solidifies the role of the multiplier in generating equivalent fractions. Students learn how equivalent fractions can be seen in the multiplication table. We can choose any two rows of the table. If we consider the numbers in one row to be numerators and the corresponding numbers in the other row to be denominators then we can make a chain of equivalent fractions.

The multipliers needed to make each fraction from the first fraction are in the top row of the table. For example, to get \(\frac{18}{30}\) from \(\frac{3}{5}\), we must use the multiplier 6, which is at the top of the column that contains both 18 and 30.

Using the multiplication table helps students answer questions like these: If I need a fraction equivalent to \(\frac{4}{7}\) with a denominator of 56, what will the numerator be? If I need a fraction equivalent to \(\frac{4}{7}\) with a numerator of 24, what will the denominator be?
In Lesson 4, students explore a variety of strategies for comparing fractions.

**Like Denominators** If two fractions have the same denominator, then they are made from the same-size unit fraction. The fraction with the greater numerator—that is, the fraction with the greater number of unit fractions—is visually larger and therefore the greater fraction.

![Example]

**Like Numerators** If two fractions have the same numerator, then they are made from the same number of unit fractions. The fraction with the lesser denominator—that is, the fraction made from the greater unit fractions—is visually larger and therefore the greater fraction.

![Example]
**Unlike Denominators** To compare two fractions with different denominators, we can rewrite them as equivalent fractions with a common denominator. For example, to compare $\frac{5}{8}$ and $\frac{7}{12}$, we can use the common denominator 24.

\[
\begin{align*}
\frac{5}{8} &= \frac{5 \times 3}{8 \times 3} = \frac{15}{24} \\
\frac{7}{12} &= \frac{7 \times 2}{12 \times 2} = \frac{14}{24}
\end{align*}
\]

Because $\frac{15}{24} > \frac{14}{24}$, we know that $\frac{5}{8} > \frac{7}{12}$.

When finding a common denominator, students consider the following three cases; however, Lesson 4 emphasizes that we can always use the product of the denominators as the common denominator.

- **One denominator is a factor of the other.** In this case, we can use the greater denominator as the common denominator. For example, for $\frac{3}{5}$ and $\frac{7}{10}$, we can use 10 as the common denominator.
- **The denominators have no common factors except 1.** In this case, we can use the product of the denominators as the common denominator. For example, for $\frac{4}{5}$ and $\frac{3}{7}$, we can use 35 as the common denominator.
- **The denominators have a common factor greater than 1.** In this case, we can find a common denominator that is less than the product of the denominators. For example, for $\frac{5}{12}$ and $\frac{4}{9}$, we can use 36 as a common denominator.

**Special Cases** Students learn that in some special situations, we can compare fractions by using reasoning.

- If both fractions are close to $\frac{1}{2}$, we can compare both to $\frac{1}{2}$. If one fraction is greater than $\frac{1}{2}$ and the other is less than $\frac{1}{2}$, then the fraction greater than $\frac{1}{2}$ is greater. For example, because $\frac{5}{8} > \frac{1}{2}$ and $\frac{2}{5} < \frac{1}{2}$, we can conclude that $\frac{5}{8} > \frac{2}{5}$.
- If both fractions are close to 1 (and both are less than 1), we can compare the fractions to 1. The fraction closer to 1 is greater. For example, $\frac{5}{6}$ is $\frac{1}{6}$ away from 1. $\frac{6}{7}$ is $\frac{1}{7}$ away from 1. Because $\frac{1}{7} < \frac{1}{6}$, $\frac{6}{7}$ is closer to 1, so it is greater.
Fractions Greater Than One The idea of building fractions from unit fractions is used to develop the ideas of fractions greater than 1 and mixed numbers. For example, the fraction \( \frac{5}{4} \) is the sum of 5 fourths.

\[
\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}
\]

Students use a variety of representations to represent \( \frac{5}{4} \) and show that it is greater than one whole.

Students can group unit fractions to show that \( \frac{5}{4} \) is one whole (\( \frac{4}{4} \)) and \( \frac{1}{4} \) more.

\[
\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1 \frac{1}{4}
\]

Convert Between Forms Students use their own methods to convert between mixed numbers and fractions. Many students will use some or all of the methods below.

\[
2 \frac{3}{4} = 2 + \frac{3}{4} = 1 + 1 + \frac{3}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = 11 \frac{4}{4}
\]

Some students will see the shortcut of multiplying \( 2 \times 4 \), to find that there are 8 fourths in 2, and then adding the 3 fourths.

To convert a fraction to a mixed number, students can reverse the thinking used above. Below is an example. Most students will not need to record all the steps.

\[
17 \frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5} = 1 + 1 + \frac{2}{5} = 3 + \frac{2}{5} = 3 \frac{2}{5}
\]
**Add and Subtract Like Mixed Numbers**

In Lesson 6, students add and subtract mixed numbers with like denominators. To add, most students will use one of the following methods:

- Add whole number parts and fraction parts separately and regroup if needed.

```
Horizontally
\[
1\frac{2}{3} + 1\frac{2}{3} = (1 + 1) + \left( \frac{2}{3} + \frac{2}{3} \right)
\]
\[
= 2\frac{4}{3}
\]
\[
= 3\frac{1}{3}
\]
```

- Vertically

```
1\frac{2}{3}

\[
+ \frac{1}{3}
\]
\[
= 2\frac{4}{3}
\]
= 3\frac{1}{3}
```

- Rewrite the mixed numbers as fractions and add.

```
1\frac{2}{3} + 1\frac{2}{3} = \frac{5}{3} + \frac{5}{3} = \frac{10}{3} = 3\frac{1}{3}
```

To subtract, students are likely to use one of these methods:

- Subtract the whole number parts and fraction parts separately, ungrouping first if needed.

```
\[
6\frac{6}{5} - 3\frac{1}{5} - 2\frac{4}{5}
\]
\[
\underline{- 2\frac{4}{5}}
\]
\[
\underline{4\frac{2}{5}}
```

- Add on from the lesser number to the greater number.

```
2\frac{4}{5} \text{ to 3} \to 7 \to 7\frac{1}{5}
\]
\[
\frac{1}{5} + 4 + \frac{1}{5} = 4\frac{2}{5}
```

- Rewrite the mixed numbers as fractions and subtract.

```
7\frac{1}{5} - 2\frac{4}{5} = \frac{36}{5} - \frac{14}{5} = \frac{22}{5} = 4\frac{2}{5}
```

Add and Subtract Unlike Fractions and Mixed Numbers

Add Unlike Fractions  In Lesson 7, students add numbers with unlike denominators. Fraction bars are used to illustrate why we must rewrite the fractions with a common denominator before adding. For example, the top bar below shows \( \frac{1}{2} + \frac{1}{3} \), but we can’t tell what the total value is. The bottom bar shows that when we express each addend as a number of sixths, we can see that the sum is \( \frac{5}{6} \).

Subtract Unlike Fractions  Students subtract unlike fractions in Lesson 8. Fraction bars are again used to illustrate why it is necessary to rewrite the fractions with a common denominator.

Find Common Denominators  Throughout Lessons 7 and 8, students use the strategies discussed in Lesson 4 to find common denominators. (See page 1EE for a list of these strategies.)
Add and Subtract Unlike Mixed Numbers  In Lessons 9 and 10, students add and subtract mixed numbers with unlike denominators. Such computations can be quite complex. They require finding equivalent fractions, adding or subtracting the whole numbers and fractions separately, and ungrouping if necessary for subtraction. Then two kinds of simplifying may be required: simplifying a fraction to the simplest form and changing a fraction greater than 1 to a mixed number, which must be added to the whole number. By sharing and discussing solution methods, students can develop effective and efficient strategies for doing these computations. The following discussion is from Lesson 9, page 69.

When presenting their solutions, pairs should describe each step, with other students asking questions or helping to clarify. Below is a discussion of Exercise 5.

Eric and Max, please explain how you solved Exercise 5.

**Eric:** First, we renamed the fractions so they had the same denominator. The denominators are 3 and 15. 3 is a factor of 15, so 15 is a common denominator. We renamed \( \frac{1}{3} \) as \( \frac{5}{15} \).

\[
4\frac{1}{3} = 4\frac{5}{15} \\
-2\frac{7}{15} = 2\frac{7}{15}
\]

**Max:** We couldn’t subtract \( \frac{7}{15} \) from \( \frac{5}{15} \) because \( \frac{5}{15} \) is smaller, so we had to ungroup. The 4 becomes a 3 because I’m giving 1 to the fifteenths. One is the same as \( \frac{15}{15} \). Adding \( \frac{15}{15} \) to \( \frac{5}{15} \), I get \( \frac{20}{15} \).

\[
3\frac{20}{15} \\
4\frac{1}{3} = 4\frac{5}{15} \\
-2\frac{7}{15} = 2\frac{7}{15}
\]

**Eric:** Then we just subtracted. We subtracted the whole numbers first: \( 3 - 2 = 1 \). Then we subtracted the fractions: \( \frac{20}{15} - \frac{7}{15} = \frac{13}{15} \).

\[
\frac{3}{15} \\
4\frac{1}{3} = 4\frac{5}{15} \\
-2\frac{7}{15} = 2\frac{7}{15}
\]

\[
\frac{13}{15}
\]
Estimation and Reasonable Answers

In Lesson 11, students discuss strategies for mentally estimating sums and differences of fractions and mixed numbers and for determining whether answers are reasonable. These strategies include the following:

- **Using benchmarks of 0, 1/2, and 1:** For example, consider $\frac{4}{9} + \frac{5}{6}$. Because $\frac{4}{9}$ is closer to $\frac{1}{2}$ than to 0 and $\frac{5}{6}$ is closer to 1 than to $\frac{1}{2}$, we know $\frac{4}{9} + \frac{5}{6}$ is close to $\frac{1}{2} + 1$, or $1\frac{1}{2}$.

- **Rounding to the nearest whole number:** For example, consider $\frac{2}{3} + \frac{5}{4}$. Because $\frac{2}{3}$ rounds to 2 and $\frac{5}{4}$ rounds to 6, the sum should be about 8.

- **Using number sense and reasoning:** For example, consider the (incorrect) equation $\frac{5}{8} + \frac{1}{6} = \frac{6}{14}$. We can reason that because one of the addends, $\frac{5}{8}$, is more than $\frac{1}{2}$, it is impossible for the sum to be less than $\frac{1}{2}$. Since $\frac{6}{14} < \frac{1}{2}$, $\frac{6}{14}$ is not a reasonable answer.

In Lesson 12, students apply what they have learned throughout the unit to solve one-step and multistep word problems involving addition and subtraction of fractions and mixed numbers. They must also describe how they know their answer is reasonable.

4. At a pizza party, the Mehta family ate $1\frac{3}{8}$ pizzas in all. They ate $\frac{9}{12}$ of a cheese pizza and some pepperoni pizza. How much pepperoni pizza did they eat?

   **Equation and answer:**
   
   $\frac{9}{12} + p = 1\frac{3}{8}$ pizzas

   **Why is the answer reasonable?**
   
   $\frac{9}{12}$ is close to 1 and $\frac{5}{8}$ is close to $\frac{1}{2}$, so the total should be close to $1\frac{1}{2}$.

Focus on Mathematical Practices

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about adding and subtracting fractions to compute the dimensions of a bird hotel.
Math Talk Learning Community

Research In the NSF research project that led to the development of *Math Expressions*, much work was done with helping teachers and students build learning communities within their classrooms. An important aspect of doing this is Math Talk. The researchers found three levels of Math Talk that go beyond the usual classroom routine of students simply solving problems and giving answers and the teacher asking questions and offering explanations. It is expected that at Grade 5, students will engage in talk at all levels.

Math Talk Level 1 A student briefly explains his or her thinking to others. The teacher helps students listen to and help others, models fuller explaining and questioning by others, and briefly probes and extends student’s ideas.

Math Talk Level 2 A student gives a fuller explanation and answers questions from other students. The teacher helps students listen to and ask good questions, models full explaining and questioning (especially for new topics), and probes more deeply to help students compare and contrast methods.

Math Talk Level 3 The explaining student manages the questioning and justifying. Students assist each other in understanding and correcting errors and in explaining more fully. The teacher monitors and assists and extends only as needed.

Summary Math Talk is important not only for discussing solutions to story problems but also for any kind of mathematical thinking students do, such as deciding how to choose an operation to solve a problem or adding and subtracting fractions.
**Beginning the Year**

**Setting Up a Learning Community**

If students have used *Math Expressions* before, ask them to describe important aspects of a math class that uses *Math Expressions*. Include the following points if the students do not raise them. If students are new to *Math Expressions*, go over these structures to begin to create a Learning Community for the year.

**Building Concepts** We understand mathematical aspects of situations (we *mathematize*) by seeing and making math drawings to visualize the mathematics and relate these to our explanations.

**Math Talk** Students explain their thinking in an instructional conversation focused on the math. Math Talk helps everyone understand better. The teacher monitors and extends student Math Talk as needed, helping students learn to talk directly to each other rather than to the teacher.

A key structure of Math Talk is called **Solve and Discuss**. The teacher selects several students to go to the board and solve a problem, using any method they choose. Students love solving at the board, and the teacher can easily see their solving process. The other students work at the same problem at their desks. Then the teacher asks the students at the board to explain their methods.

An explaining student relates his or her solution to a math drawing, asks if there are any questions, and responds to questions. Listeners are encouraged to ask questions when they do not understand and also to ask lead-in questions that focus on the mathematics that are key to understanding the solution to the problem. Students can also suggest edits to the explanation or solution. So this is actually a Solve, Explain, Question, and Justify structure called Solve and Discuss for short.

During Solve and Discuss, teachers can stand at the side or back of the classroom to help students interact more directly with each other. Teachers say that it is necessary for them to “bite their tongue” to keep from doing all of the talking; student voices and explanations will emerge if you wait. For new topics, teachers may need to model explaining so that students learn to use new vocabulary, but some students can explain even for a new topic.

Usually only two students will explain their solution during Solve and Discuss because listening to many similar explanations is not fruitful. It is better to go on to the next problem to engage students in a new situation to Solve and Discuss. If there are particularly interesting solutions or someone at the board needs a turn at explaining, more than two students should explain their solutions.

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**Learning Community—Best Practices**

Aspire to make your classroom a place where all students listen to understand one another. Explain to students that this is different from just being quiet when someone else is talking. This involves thinking about what a person is saying so that you could explain it yourself or help them explain it more clearly. Also, students need to listen so that they can ask a question or help the explainer. Listening can also help them learn more about a concept.
Helping Community Everyone is a learner and a teacher and helps each other understand and explain. This is a safe community in which there is no making fun of anyone ever. A student can always ask for help, and it is brave to ask for help or say, I don’t understand. Everyone makes mistakes, and we thank people who make mistakes because it helps us all learn how to recognize and correct mistakes.

Quick Practice Quick Practice starts each lesson. These activities help everyone become fluent in core content. Student Leaders usually lead the Quick Practice.

Student Leaders Everyone is expected to be a leader in this class. Volunteers will lead Quick Practices at first, and eventually everyone will get a turn. Student Leaders also help their classmates understand or explain concepts and skills; everyone can be a Student Leader in class discussions or in small group or pair work.

In the first Quick Practice of Unit 1, students practice writing and comparing fractions. Six Student Leaders will lead the class in writing six fractions that are less than \( \frac{1}{2} \).

Posters The Grade 5 kit includes posters.
- Place Value Poster for reading and writing decimals and mixed decimals.
- Fraction Poster showing fraction models and operations with fractions.
- Geometry Poster showing the hierarchy of properties of two dimensional shapes.
The Learning Community (continued)

MathBoards Students use a MathBoard as a tool for recording their thinking, relating their drawings to math language, and justifying their reasoning.

Math Drawings These are drawings that focus on the mathematical aspects of a situation or of quantities. They may be generated by the student or teacher, or they may be research-based forms introduced in Math Expressions to help students understand and represent mathematical concepts, situations, and notation. Correct mathematical notation and vocabulary are always used in solutions and explanations, and these are related to the math drawings. But students also can use their own meaningful language to clarify concepts and make them memorable.

Mathematical Practices A Math Talk Learning Community often uses Mathematical Practices 1 through 7 in a Solve and Discuss of a problem. Mathematical Practice 8 often is used as students reason across different problems. Helping students consistently and clearly explain their reasoning, and relate different solutions for one problem or use the same solution across different problems, fosters all of the Mathematical Practices.