Interwoven Strands of Proficiency
Conceptual Understanding
Procedural Fluency
Strategic Competence
Adaptive Reasoning
Productive Disposition

(National Research Council, 2001)

PROBLEM SOLVING

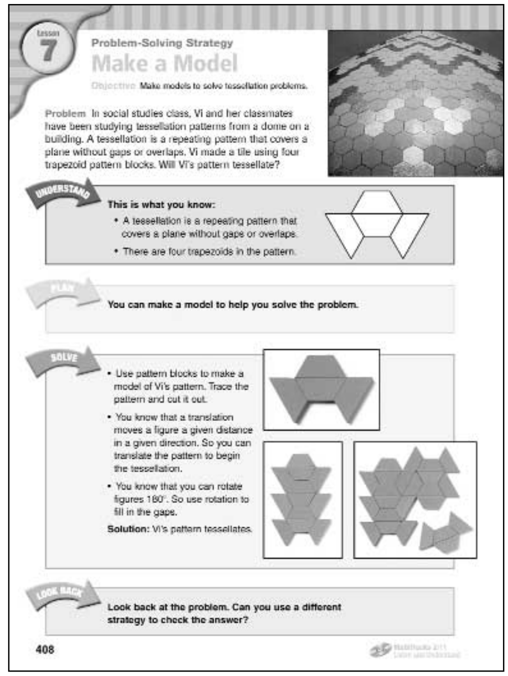
The problem-solving strand in ***Houghton Mifflin Math*** involves both solving problems and formulating problems. This aligns with the definition of strategic competence used by the authors of *Adding It Up.*

> *Strategic competence* refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is similar to what has been called problem solving and problem formulation in the literature of mathematics education and cognitive science (NRC, 2001, p. 124).

This section will address both problem *solving* and problem *formulation.*

Solving problems is an integral part of ***Houghton Mifflin Math.*** Topics are often introduced with word problems and instructional lessons always include word problems. In addition, there are lessons designed to teach specific problem-solving skills and strategies and there are many opportunities for students to solve problems involving real-world data. Studies show that students' success in solving problems is related to the amount of time they spend on problem solving.

> Fourth grade students who reported doing math problems from the textbook every day scored the highest on the 2000 National Assessment of Educational Progress (NAEP, 2001).



Lesson 7

Problem-Solving Strategy

Make a Model

Objective Make models to solve tessellation problems.

Problem In social studies class, Vi and her classmates have been studying tessellation patterns from a dome on a building. A tessellation is a repeating pattern that covers a plane without gaps or overlaps. Vi made a tile using four trapezoid pattern blocks. Will Vi's pattern tessellate?

UNDERSTAND

This is what you know:

- A tessellation is a repeating pattern that covers a plane without gaps or overlaps.
- There are four trapezoids in the pattern.

PLAN

You can make a model to help you solve the problem.

SOLVE

- Use pattern blocks to make a model of Vi's pattern. Trace the pattern and cut it out.
- You know that a translation moves a figure a given distance in a given direction. So you can translate the pattern to begin the tessellation.
- You know that you can rotate figures 180°. So use rotation to fill in the gaps.

Solution: Vi's pattern tessellates.

LOOK BACK

Look back at the problem. Can you use a different strategy to check the answer?

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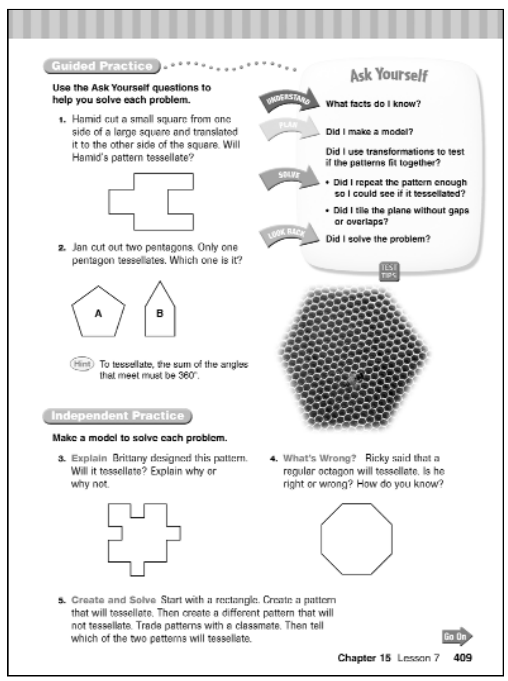

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. Hamid cut a small square from one side of a large square and translated it to the other side of the square. Will Hamid's pattern tessellate?

2. Jan cut out two pentagons. Only one pentagon tessellates. Which one is it?

A B

Hint To tessellate, the sum of the angles that meet must be 360°.

Ask Yourself

UNDERSTAND What facts do I know?

PLAN Did I make a model?

Did I use transformations to test if the patterns fit together?

SOLVE

- Did I repeat the pattern enough so I could see if it tessellated?
- Did I tile the plane without gaps or overlaps?

LOOK BACK Did I solve the problem?

TEST TIPS

Independent Practice

Make a model to solve each problem.

3. Explain Brittany designed this pattern. Will it tessellate? Explain why or why not.

4. What's Wrong? Ricky said that a regular octagon will tessellate. Is he right or wrong? How do you know?

5. Create and Solve Start with a rectangle. Create a pattern that will tessellate. Then create a different pattern that will not tessellate. Trade patterns with a classmate. Then tell which of the two patterns will tessellate.

Go On

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The problem-solving instruction in ***Houghton Mifflin Math*** is based on the four-step problem-solving method of George Polya (1957): Understand, Plan, Solve, and Look Back. These four steps are used in the lesson development section of problem-solving lessons.

Polya (1957) also generated a list of strategies that can be used to solve problems. This list is similar to the list of strategies used in ***Houghton Mifflin Math.***

The scope and sequence for problem-solving strategies in ***Houghton Mifflin Math*** is shown in the chart below.

The chart illustrates how the strategies are developed from kindergarten through grade 6.

> …no strategy is learned once and for all; strategies are learned over time, are applied in particular contexts, and become more refined, elaborate, and flexible as they are used in increasingly complex problem situations (NCTM, 2000, p. 53).

The National Council of Teachers of Mathematics (2000) recommends direct teaching of problem-solving strategies as well as inclusion of numerous opportunities for students to choose their own strategies.

> …strategies must receive instructional attention if students are expected to learn them… Opportunities to use strategies must be embedded naturally in the curriculum across the content areas. By the time students reach the middle grades, they should be skilled at recognizing when various strategies are appropriate to use and should be capable of deciding when and how to use them (NCTM, 2000, p. 53).

In addition to strategy lessons, students using ***Houghton Mifflin Math*** are given numerous opportunities to choose strategies and discuss which strategies they used.

Strategies	K	1	2	3	4	5	6
Act it out with models	●	●	●	●	▲	▲	▲
Choose a method		●	●	●	▲	▲	▲
Choose an operation	●	●	●	▲	▲	▲	▲
Draw a picture or diagram	●	●	●	●	●	▲	▲
Find a pattern	●	●	●	▲	▲	▲	▲
Guess and check	●	●	●	●	▲	▲	▲
Make a model	●	●	●	●	●	●	▲
Make a table or chart	●	●	●	●	▲	▲	▲
Make an organized list			●	●	●	▲	▲
Monitor and reflect on the process	●	●	●	●	▲	▲	▲
Solve a simpler problem				●	●	▲	▲
Use a formula					●	●	▲
Use logical reasoning	●	●	●	●	▲	▲	▲
Work backward				●	●	▲	▲
Write a number sentence or equation	●	●	●	●	●	▲	▲

KEY Teach and Apply ● Practice and Apply ▲

Research also shows that successful problem solving involves learning how to *monitor* and *reflect* on the process.

> Students should have frequent opportunities to…solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking (NCTM, 2000, p. 51).

Research (Garofalo and Lester, 1985; Schoenfeld, 1987) indicates that students' problem-solving failures are often due not to a lack of mathematical knowledge but to the ineffective use of what they do know. Good problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems (Bransford et al., 1999).

As the following examples illustrate, beginning in kindergarten, students using ***Houghton Mifflin Math*** are given numerous opportunities that require them to self-assess and adjust their strategies.

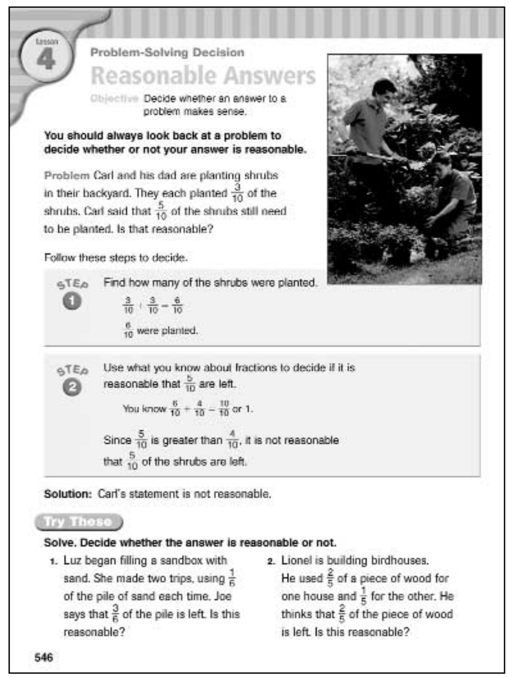
In kindergarten, students are given this problem orally.

> …draw a blue basket that holds more than a red basket. Draw a green basket that holds less than the red basket. Circle the basket that holds the most (***Houghton Mifflin Math***, grade K, p. 234).

This problem will require students to self-assess their work and adjust if necessary.

Problems that require students to consider whether or not an answer is reasonable help them develop the necessary mindset needed to self-assess and adjust. Consider this problem from grade 3.

> Carl and his dad are planting shrubs in their backyard. They each planted $3/10$ of the shrubs. Carl said that $5/10$ of the shrubs still need to be planted. Is that reasonable? (***Houghton Mifflin Math,*** grade 3, p. 546).



Lesson 4

Problem-Solving Decision

Reasonable Answers

Objective Decide whether an answer to a problem makes sense.

You should always look back at a problem to decide whether or not your answer is reasonable.

Problem Carl and his dad are planting shrubs in their backyard. They each planted $\frac{3}{10}$ of the shrubs. Carl said that $\frac{5}{10}$ of the shrubs still need to be planted. Is that reasonable?

Follow these steps to decide.

STEP 1 Find how many of the shrubs were planted.

$\frac{3}{10} + \frac{3}{10} = \frac{6}{10}$

$\frac{6}{10}$ were planted.

STEP 2 Use what you know about fractions to decide if it is reasonable that $\frac{5}{10}$ are left.

You know $\frac{6}{10} + \frac{4}{10} = \frac{10}{10}$ or 1.

Since $\frac{5}{10}$ is greater than $\frac{4}{10}$, it is not reasonable that $\frac{5}{10}$ of the shrubs are left.

Solution: Carl's statement is not reasonable.

Try These

Solve. Decide whether the answer is reasonable or not.

1. Luz began filling a sandbox with sand. She made two trips, using $\frac{1}{6}$ of the pile of sand each time. Joe says that $\frac{3}{6}$ of the pile is left. Is this reasonable?
2. Lionel is building birdhouses. He used $\frac{2}{5}$ of a piece of wood for one house and $\frac{1}{5}$ for the other. He thinks that $\frac{2}{5}$ of the piece of wood is left. Is this reasonable?

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Student Book, grade 3, page 546

Students in grade 5 are shown how self-monitoring and adjustment can be used in solving this problem.

> For the past 3 years, the Antique Automobile Club's show has averaged 880 tickets sold per year. Ticket sales are expected to be about the same this year. If the show cost $30,000 to put on, will a ticket price of $35 be enough to cover the costs? Explain your answer (***Houghton Mifflin Math,*** grade 5, p. 80).

Students find out that using estimation

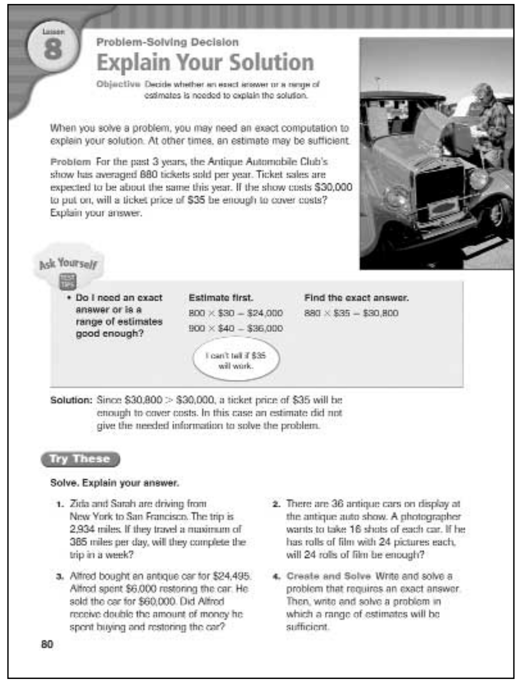
> 800 × $30 = $24,000
> 900 × $40 = $36,000

is not sufficient to solve the problem. They must adjust their thinking and multiply $35 by 880.

Throughout the grades, students are frequently asked to do a calculation and then use estimation or inverse operations to check if their answer is reasonable. This also gives students opportunities to self-assess and adjust their strategies.

The examples are also useful for illustrating that strategic competence is not independent; it is intertwined with the other strands of mathematical proficiency.

> There are mutually supportive relations between strategic competence and both conceptual understanding and procedural fluency (NRC, 2001, p. 127).



Lesson 8

Problem-Solving Decision

Explain Your Solution

Objective Decide whether an exact answer or a range of estimates is needed to explain the solution.

When you solve a problem, you may need an exact computation to explain your solution. At other times, an estimate may be sufficient.

Problem For the past 3 years, the Antique Automobile Club's show has averaged 880 tickets sold per year. Ticket sales are expected to be about the same this year. If the show costs $30,000 to put on, will a ticket price of $35 be enough to cover costs? Explain your answer.

Ask Yourself

• **Do I need an exact answer or is a range of estimates good enough?**	**Estimate first.** 800 × $30 = $24,000 900 × $40 = $36,000 I can't tell if $35 will work.	**Find the exact answer.** 880 × $35 = $30,800

Solution: Since $30,800 > $30,000, a ticket price of $35 will be enough to cover costs. In this case an estimate did not give the needed information to solve the problem.

Try These

Solve. Explain your answer.

1. Zida and Sarah are driving from New York to San Francisco. The trip is 2,934 miles. If they travel a maximum of 385 miles per day, will they complete the trip in a week?
2. There are 36 antique cars on display at the antique auto show. A photographer wants to take 16 shots of each car. If he has rolls of film with 24 pictures each, will 24 rolls of film be enough?
3. Alfred bought an antique car for $24,495. Alfred spent $6,000 restoring the car. He sold the car for $60,000. Did Alfred receive double the amount of money he spent buying and restoring the car?
4. **Create and Solve** Write and solve a problem that requires an exact answer. Then, write and solve a problem in which a range of estimates will be sufficient.

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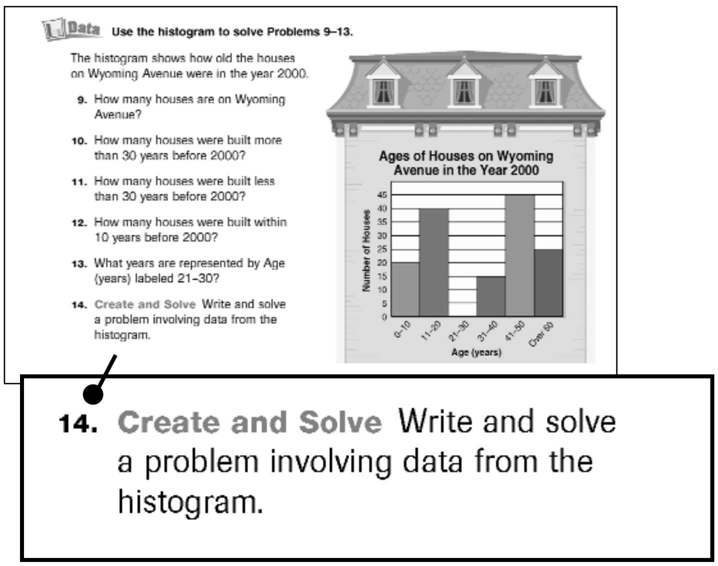
Student Book, grade 5, page 80

Students using ***Houghton Mifflin Math*** have many opportunities for problem *formulation.* Beginning in grade 1, many exercises require students to write story problems. These exercises are highlighted in the program with the title **Create Your Own** or **Create and Solve.**



Data Use the histogram to solve Problems 9–13.

The histogram shows how old the houses on Wyoming Avenue were in the year 2000.

9. How many houses are on Wyoming Avenue?
10. How many houses were built more than 30 years before 2000?
11. How many houses were built less than 30 years before 2000?
12. How many houses were built within 10 years before 2000?
13. What years are represented by Age (years) labeled 21–30?
14. Create and Solve Write and solve a problem involving data from the histogram.

14. Create and Solve Write and solve a problem involving data from the histogram.

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Name

Create and Solve

Write a subtraction story that compares the birds.

Write the subtraction sentence.

Tell a story to match the number sentence. 7 − 4 = 3

Draw a picture to show your story.

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