PROBLEM SOLVING

The problem-solving strand in *Houghton Mifflin Math* involves both solving problems and formulating problems. This aligns with the definition of strategic competence used by the authors of *Adding It Up*.

*Strategic competence* refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is similar to what has been called problem solving and problem formulation in the literature of mathematics education and cognitive science (NRC, 2001, p. 124).

This section will address both problem solving and problem formulation.

Solving problems is an integral part of *Houghton Mifflin Math*. Topics are often introduced with word problems and instructional lessons always include word problems. In addition, there are lessons designed to teach specific problem-solving skills and strategies and there are many opportunities for students to solve problems involving real-world data. Studies show that students’ success in solving problems is related to the amount of time they spend on problem solving.

Fourth grade students who reported doing math problems from the textbook every day scored the highest on the 2000 National Assessment of Educational Progress (NAEP, 2001).
The problem-solving instruction in *Houghton Mifflin Math* is based on the four-step problem-solving method of George Polya (1957): Understand, Plan, Solve, and Look Back. These four steps are used in the lesson development section of problem-solving lessons.

Polya (1957) also generated a list of strategies that can be used to solve problems. This list is similar to the list of strategies used in *Houghton Mifflin Math*.

The scope and sequence for problem-solving strategies in *Houghton Mifflin Math* is shown in the chart below.

The chart illustrates how the strategies are developed from kindergarten through grade 6.

...no strategy is learned once and for all; strategies are learned over time, are applied in particular contexts, and become more refined, elaborate, and flexible as they are used in increasingly complex problem situations (NCTM, 2000, p. 53).

The National Council of Teachers of Mathematics (2000) recommends direct teaching of problem-solving strategies as well as inclusion of numerous opportunities for students to choose their own strategies.

...strategies must receive instructional attention if students are expected to learn them... Opportunities to use strategies must be embedded naturally in the curriculum across the content areas. By the time students reach the middle grades, they should be skilled at recognizing when various strategies are appropriate to use and should be capable of deciding when and how to use them (NCTM, 2000, p. 53).

In addition to strategy lessons, students using *Houghton Mifflin Math* are given numerous opportunities to choose strategies and discuss which strategies they used.
Research also shows that successful problem solving involves learning how to monitor and reflect on the process.

Students should have frequent opportunities to solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking (NCTM, 2000, p. 51).

Research (Garofalo and Lester, 1985; Schoenfeld, 1987) indicates that students’ problem-solving failures are often due not to a lack of mathematical knowledge but to the ineffective use of what they do know. Good problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems (Bransford et al., 1999).

As the following examples illustrate, beginning in kindergarten, students using *Houghton Mifflin Math* are given numerous opportunities that require them to self-assess and adjust their strategies.

In kindergarten, students are given this problem orally:

…draw a blue basket that holds more than a red basket. Draw a green basket that holds less than the red basket. Circle the basket that holds the most (*Houghton Mifflin Math*, grade K, p. 234).

This problem will require students to self-assess their work and adjust if necessary.

Problems that require students to consider whether or not an answer is reasonable help them develop the necessary mindset needed to self-assess and adjust. Consider this problem from grade 3.

Carl and his dad are planting shrubs in their backyard. They each planted \( \frac{3}{10} \) of the shrubs. Carl said that \( \frac{7}{10} \) of the shrubs still need to be planted. Is that reasonable? (*Houghton Mifflin Math*, grade 3, p. 546).
Students in grade 5 are shown how self-monitoring and adjustment can be used in solving this problem.

For the past 3 years, the Antique Automobile Club's show has averaged 880 tickets sold per year. Ticket sales are expected to be about the same this year. If the show cost $30,000 to put on, will a ticket price of $35 be enough to cover the costs? Explain your answer (Houghton Mifflin Math, grade 5, p. 80).

Students find out that using estimation

\[800 \times $30 = $24,000\]
\[900 \times $40 = $36,000\]

is not sufficient to solve the problem. They must adjust their thinking and multiply $35 by 880.

Throughout the grades, students are frequently asked to do a calculation and then use estimation or inverse operations to check if their answer is reasonable. This also gives students opportunities to self-assess and adjust their strategies.

The examples are also useful for illustrating that strategic competence is not independent; it is intertwined with the other strands of mathematical proficiency.

There are mutually supportive relations between strategic competence and both conceptual understanding and procedural fluency (NRC, 2001, p. 127).
Students using *Houghton Mifflin Math* have many opportunities for problem formulation. Beginning in grade 1, many exercises require students to write story problems. These exercises are highlighted in the program with the title *Create Your Own* or *Create and Solve*.

**14. Create and Solve** Write and solve a problem involving data from the histogram.