Interwoven Strands of Proficiency

- Conceptual Understanding
- **Procedural Fluency**
- Strategic Competence
- Adaptive Reasoning
- Productive Disposition

(National Research Council, 2001)

ALGORITHMS

Knuth (1974) defines an algorithm as "a precisely defined sequence of rules telling how to produce specified output information in a finite number of steps." Steen (1990) puts this more simply by defining an algorithm as "a recipe for computation."

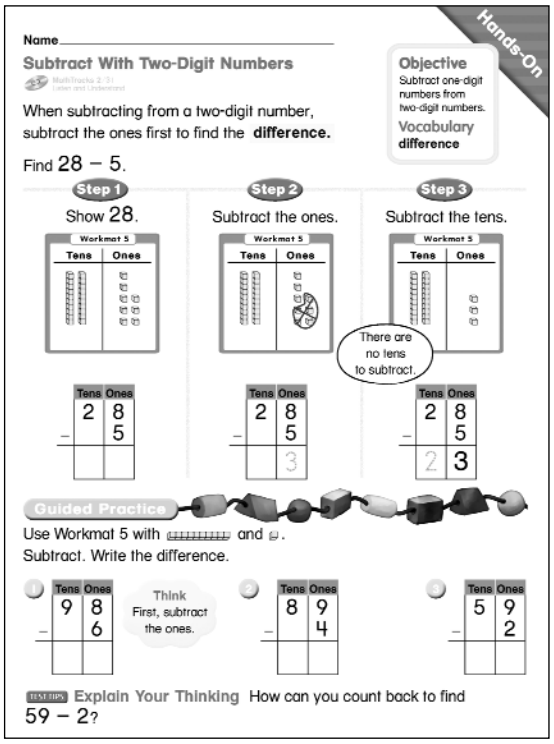
The learning of mathematical algorithms is an important component of *procedural fluency,* the ability to perform procedures flexibly, accurately, and efficiently (NRC, 2001, p. 162).

There is a consensus from research that students need to understand how algorithms work. Conceptual understanding and procedural fluency are linked.

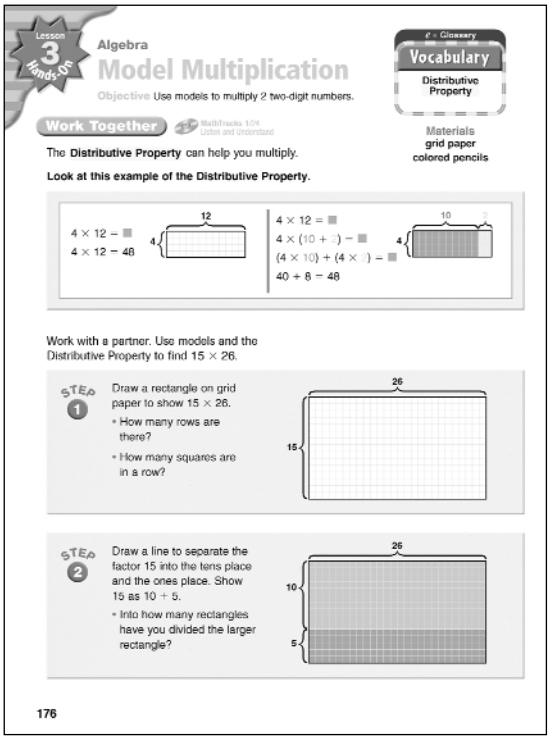
> When the initial computational procedures that students use to solve multidigit problems reflect their understanding of numbers, understanding and fluency develop together (NRC, 2001, p. 196).

> …the Chinese teachers often cited an old saying to introduce further discussions of an algorithm. "Know how, and also know why." In adapting this saying which encourages people to discover a reason behind an action, the teachers gave it new and specific meaning *—to know how to carry out an algorithm and to know why it makes sense mathematically* (Ma, 1999, p. 108).

In ***Houghton Mifflin Math,*** concrete models are used to show students how the algorithms work. For example, in grade 1, base-ten blocks are used to introduce students to the algorithm for subtracting two-digit numbers. In grade 4, the algorithm for multiplying 2 two-digit numbers is developed using an array model and the Distributive Property.



Student Book, grade 1, page 627



Student Book, grade 4, page 176

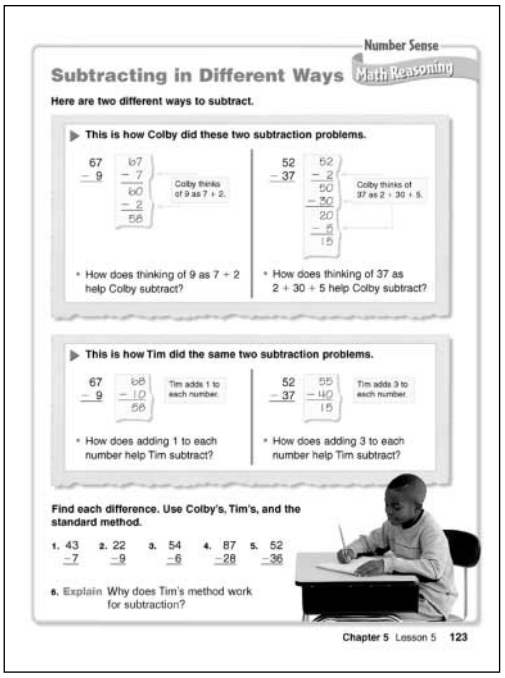
Other research about algorithms focuses on whether students should be taught specific algorithms or whether they should be allowed to invent their own.

> …many students are likely to need help learning efficient forms of multidigit procedures. …children can and do devise or invent algorithms for carrying out multidigit computations (NRC, 2001, p. 197).

> Instruction should not be based on extreme positions that students learn, on the one hand, solely by internalizing what a teacher or book says or, on the other hand, solely by inventing mathematics on their own (NRC, 2001, p. 11).

Based on this research, ***Houghton Mifflin Math*** presents specific algorithms, but also discusses alternative algorithms and encourages students to invent their own algorithms. An example can be seen in grade 3, page 123, where alternative subtraction algorithms are explored.

Further, students learn that algorithms have changed over time. In Europe in the 1400s, Gelosia Multiplication was popular. This is shown in grade 4, page 197.


Number Sense

Subtracting in Different Ways Math Reasoning

Here are two different ways to subtract.

This is how Colby did these two subtraction problems.

67 − 9 = 58 (Colby thinks of 9 as 7 + 2.)

52 − 37 = 15 (Colby thinks of 37 as 2 + 30 + 5.)

- How does thinking of 9 as 7 + 2 help Colby subtract?
- How does thinking of 37 as 2 + 30 + 5 help Colby subtract?

This is how Tim did the same two subtraction problems.

67 − 9 = 58 (Tim adds 1 to each number.)

52 − 37 = 15 (Tim adds 3 to each number.)

- How does adding 1 to each number help Tim subtract?
- How does adding 3 to each number help Tim subtract?

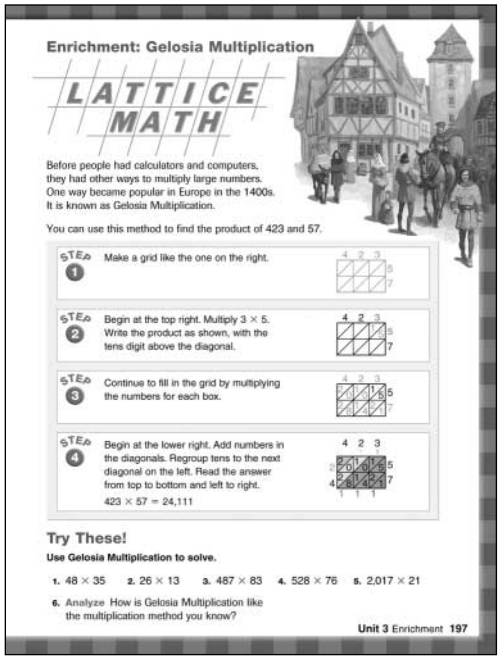
Find each difference. Use Colby's, Tim's, and the standard method.

1. 43 − 7
2. 22 − 9
3. 54 − 6
4. 87 − 28
5. 52 − 36

6. Explain Why does Tim's method work for subtraction?

Chapter 5 Lesson 5 123

Student Book, grade 3, page 123


Enrichment: Gelosia Multiplication

LATTICE MATH

Before people had calculators and computers, they had other ways to multiply large numbers. One way became popular in Europe in the 1400s. It is known as Gelosia Multiplication.

You can use this method to find the product of 423 and 57.

STEP 1 Make a grid like the one on the right.

STEP 2 Begin at the top right. Multiply 3 × 5. Write the product as shown, with the tens digit above the diagonal.

STEP 3 Continue to fill in the grid by multiplying the numbers for each box.

STEP 4 Begin at the lower right. Add numbers in the diagonals. Regroup tens to the next diagonal on the left. Read the answer from top to bottom and left to right.

423 × 57 = 24,111

Try These!

Use Gelosia Multiplication to solve.

1. 48 × 35
2. 26 × 13
3. 487 × 83
4. 528 × 76
5. 2,017 × 21

6. Analyze How is Gelosia Multiplication like the multiplication method you know?

Unit 3 Enrichment 197

Student Book, grade 4, page 197