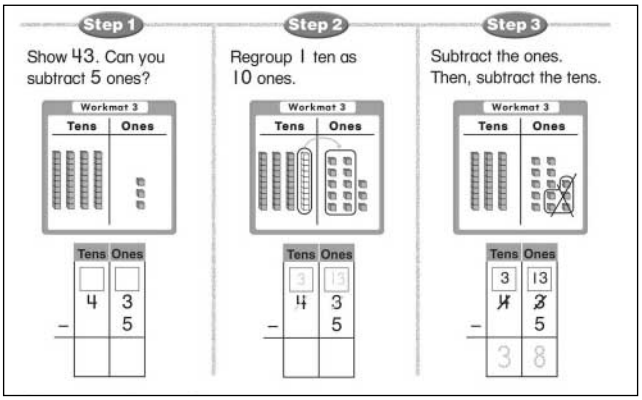
Basic Operations

During the elementary grades, children need to develop what Van de Walle (2001) terms *operation sense,* a highly integrated understanding of the four operations and the many different but related meanings these operations take on in real contexts.

> For all four operations, word problems and models (usually counters and number lines) are the two basic tools the teacher has to help students develop operation concepts. Models serve both as a tinker toy to analyze what the structure of the story problem may be and also a connection between the meaning in the story problem and the symbolic equation (Van de Walle, 2001, pp. 107–108).

Houghton Mifflin Math gives students numerous opportunities to make connections between a subtraction problem, models, and symbolic notation. For example, in grade 2 when solving a problem requiring subtraction with regrouping, students see the problem worked out both with models and using the symbolic notation. This parallel presentation helps students make the connections.



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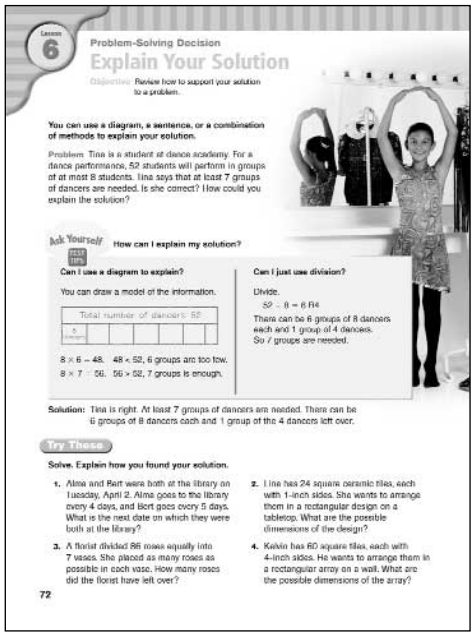
After students have had ample experience connecting problems, models, and symbolic notation, the goal is for students to be able to make these connections themselves by choosing a way to solve the problem that makes sense to them. Further, students need to be able to explain how they solved the problem.



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> Forcing children to write exactly the equation we have in mind or to solve a problem as we would solve it may be a waste of precious classroom time. The time may better be spent allowing children to solve problems and write equations in ways that make the most sense to them and discussing these solutions to share and develop a rich array of ideas (Gutstein and Romberg, 1995).

It is important that the above statement not be used to preclude the use of directed activities with models and word problems. Students need to be shown the relationship between models, operations, and word problems before they can make connections on their own.

Houghton Mifflin Math gives students many opportunities to choose their own methods for solving problems, but this occurs after students have had experiences watching the teacher model varied types of problems.

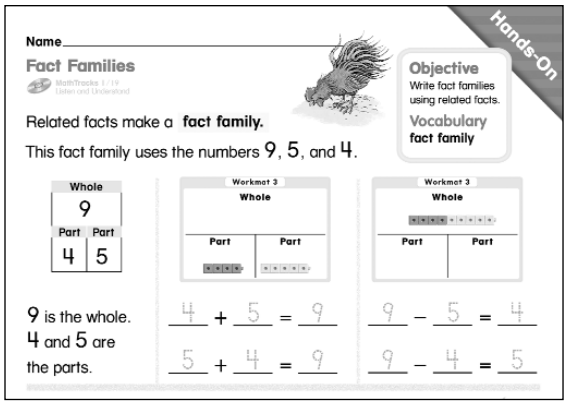
To develop operation sense, it is also necessary to connect operations.

> …it is very important to connect addition and subtraction and to connect multiplication and division.…models form the connection much better than words. The models for subtraction are precisely the same as those for addition. Similarly, the models for multiplication and division are alike. It is only a matter of which numbers are the unknown in the problem (Van de Walle, 2001, p. 109).
>
> When students have a model for one operation and have written an equation to go with it, it is often useful to ask what other equation can be written for the same model (Van de Walle, 2001, p. 108).

The use of part-part-whole mats and fact families in ***Houghton Mifflin Math*** help make the connections between related operations.

Van de Walle (2001, p. 109) describes models, words, and symbols as three separate languages and suggests that students make translations from one language to another. He gives the following example.

> Students could be asked to make up a story problem to go with the equation 4 × 7 = 28 and to illustrate the meaning with a drawing.



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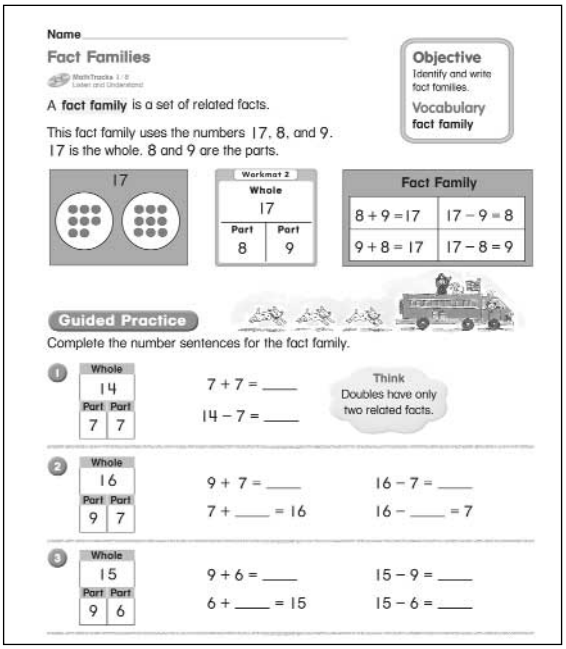

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In ***Houghton Mifflin Math***, students often make up story problems to go with number sentences.

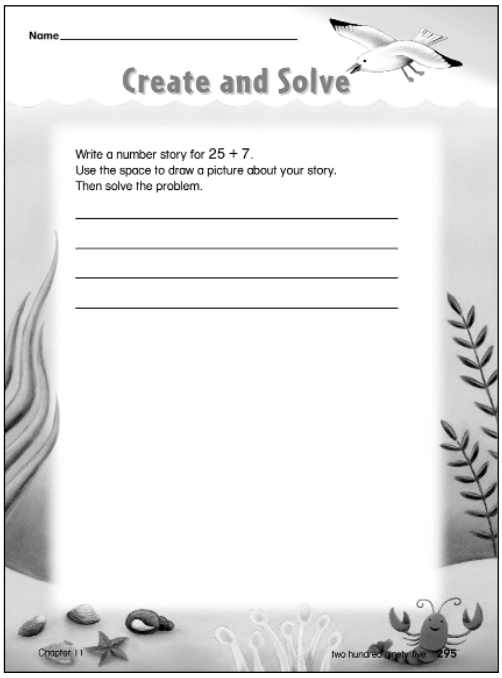
In order for children to know how to solve word problems involving the four basic operations, they must be exposed to the many different types of problems associated with each operation.

> Students should be able to decide whether to add, subtract, multiply, or divide for a particular problem. To do so, they must recognize that the same operation can be applied in problem situations that on the surface seem quite different from one another… (NCTM, 2000, p. 33).

Researchers identify four different categories of addition and subtraction problems: join problems, separate problems, part-part-whole problems, and compare problems (Carpenter and Moser, 1983; Gutstein and Romberg, 1995; Fennema, Carpenter, Levi, Franke, and Empson, 1997).

Some textbook programs tend to focus on join and separate problems.

> In traditional textbook programs, there is an almost exclusive emphasis on join and change (separate) problems with result unknown… The reasons for this are teachers' reluctance to let children wrestle with a difficult problem and the fact that they themselves learned that the result-unknown problems "define" addition and subtraction. This emphasis on the two easiest forms of problems is a serious curricular error that causes difficulty as children face all of these problem structures and as they encounter larger numbers in the upper grades (Van de Walle, 2001, p. 111).


Name _____

Create and Solve

Write a number story for 25 + 7.
Use the space to draw a picture about your story.
Then solve the problem.

_____

_____

_____

_____

Chapter 11

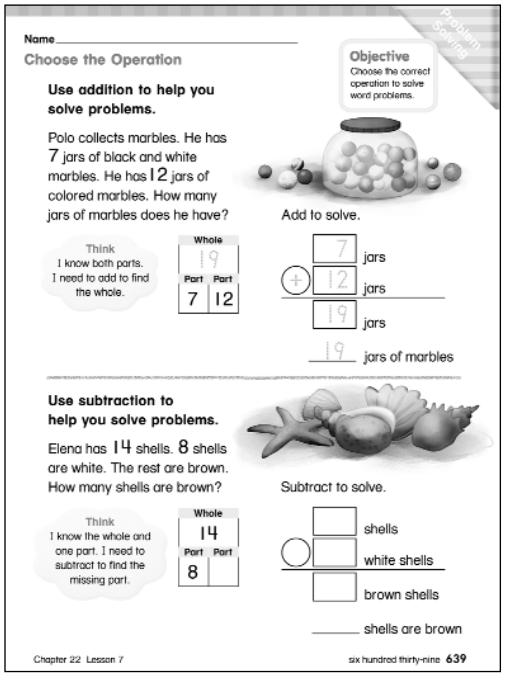
two hundred ninety-five 295

Student Book, grade 2, page 295

The authors of ***Houghton Mifflin Math*** have taken great care to be sure students using the program are exposed to all four categories of problems. As early as grade 1, students learn about addition and subtraction problems that involve comparisons or the part-part-whole model.

Similarly, researchers have identified four classes of multiplicative structures: equal groups, multiplicative comparison, combinations, and product-of-measures problems (Greer, 1992).

> Of these four classes, combinations and product-of-measures problems receive much less attention (Van de Walle, 2001, p. 115). Again, the authors of ***Houghton Mifflin Math*** include problems from all four classes. Combinations are introduced informally in grade 3 and then developed formally later. An introduction to area problems (products-of-measures problems) also begins in grade 3. In grade 4, the area model is used to help illustrate the Distributive Property.



Name _____

Choose the Operation

Objective
Choose the correct operation to solve word problems.

Use addition to help you solve problems.

Polo collects marbles. He has 7 jars of black and white marbles. He has 12 jars of colored marbles. How many jars of marbles does he have?

Add to solve.

Think
I know both parts. I need to add to find the whole.

Whole 19
Part 7 Part 12

7 jars
+ 12 jars
19 jars
19 jars of marbles

Use subtraction to help you solve problems.

Elena has 14 shells. 8 shells are white. The rest are brown. How many shells are brown?

Subtract to solve.

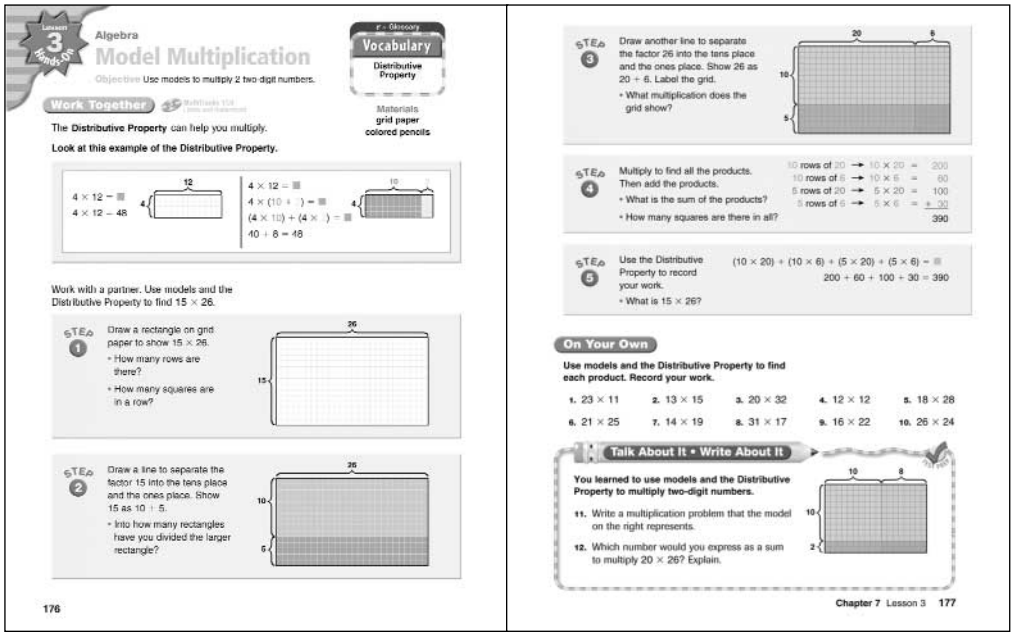
Think
I know the whole and one part. I need to subtract to find the missing part.

Whole 14
Part 8 Part ?

_____ shells
– _____ white shells
_____ brown shells
_____ shells are brown

Chapter 22 Lesson 7 six hundred thirty-nine 639

Student Book, grade 1, page 639



Lesson 3 Hands On

Algebra

Model Multiplication

Objective Use models to multiply 2 two-digit numbers.

Vocabulary
Distributive Property

Materials
grid paper
colored pencils

Work Together

The **Distributive Property** can help you multiply.

Look at this example of the Distributive Property.

$4 \times 12 = \blacksquare$
$4 \times 12 = 48$

$4 \times 12 = \blacksquare$
$4 \times (10 + \ ) = \blacksquare$
$(4 \times 10) + (4 \times \ ) = \blacksquare$
$40 + 8 = 48$

Work with a partner. Use models and the Distributive Property to find 15×26.

STEP 1 Draw a rectangle on grid paper to show 15×26.
- How many rows are there?
- How many squares are in a row?

STEP 2 Draw a line to separate the factor 15 into the tens place and the ones place. Show 15 as $10 + 5$.
- Into how many rectangles have you divided the larger rectangle?

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STEP 3 Draw another line to separate the factor 26 into the tens place and the ones place. Show 26 as $20 + 6$. Label the grid.
- What multiplication does the grid show?

STEP 4 Multiply to find all the products. Then add the products.
- What is the sum of the products?
- How many squares are there in all?

10 rows of 20 → $10 \times 20 = 200$
10 rows of 6 → $10 \times 6 = 60$
5 rows of 20 → $5 \times 20 = 100$
5 rows of 6 → $5 \times 6 = + 30$
390

STEP 5 Use the Distributive Property to record your work.
- What is 15×26?

$(10 \times 20) + (10 \times 6) + (5 \times 20) + (5 \times 6) = \blacksquare$
$200 + 60 + 100 + 30 = 390$

On Your Own

Use models and the Distributive Property to find each product. Record your work.

1. 23×11
2. 13×15
3. 20×32
4. 12×12
5. 18×28
6. 21×25
7. 14×19
8. 31×17
9. 16×22
10. 26×24

Talk About It • Write About It

You learned to use models and the Distributive Property to multiply two-digit numbers.

11. Write a multiplication problem that the model on the right represents.
12. Which number would you express as a sum to multiply 20×26? Explain.

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Student Book, grade 4, pages 176–177