

Interwoven Strands of Proficiency

Conceptual Understanding

Procedural Fluency
Strategic Competence
Adaptive Reasoning
Productive Disposition

(National Research Council, 2001)

CONCEPT DEVELOPMENT

Just as the strands of mathematical competency are interwoven, there are many connections between mathematical topics.

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful (NRC, 2001, p. 118).

Knowledge that has been learned with understanding provides the basis for generating new knowledge and for solving new and unfamiliar problems (Bransford, Brown, and Cocking, 1999).

A coherent curriculum effectively organizes and integrates important mathematical ideas so that students can see how the ideas build on, or connect with, other ideas, thus enabling them to develop new understandings and skills (NCTM, 2000).

Enabling students to integrate mathematical ideas is a major goal of the authors of *Houghton Mifflin Math*.

Guided Practice

Find the sum.

Think
I can use
 $8 + 8 = 16$
to help me find
 $8 + 9$ and
 $9 + 8$.

$8 + 8 = 16$ $8 + 9 = \underline{\quad}$ $9 + 8 = \underline{\quad}$

$7 + 7 = \underline{\quad}$ $7 + 8 = \underline{\quad}$ $8 + 7 = \underline{\quad}$

TEST TIP Explain Your Thinking How does using $4 + 4 = 8$ help you solve $4 + 5$?

Student Book, grade 2, page 31

For example, on page 31 in grade 2, students use their knowledge of doubles and the Commutative Property of Addition to learn addition facts. This type of connection is substantiated by the authors of *Adding It Up*:

As an example of how a knowledge cluster can make learning easier, consider the cluster students might develop for adding whole numbers. If students understand that addition is commutative (e.g., $3 + 5 = 5 + 3$), their learning of basic addition combinations is reduced by almost half. By exploiting their knowledge of other relationships such as that between the doubles (e.g., $5 + 5$ and $6 + 6$) and other sums, they can reduce still further the number of addition combinations they need to learn (NRC, 2001, p. 120).

For another example of mathematical connections in *Houghton Mifflin Math*, see page 152 in grade 4. The concept of multiplying a two-digit number by a one-digit number is connected to the concept of place value and multiplication of one-digit numbers. Again, this aligns with the mathematical research.

Lesson 4
Multiply Two-Digit Numbers by One-Digit Numbers
 Objective: Regroup ones or tens to multiply.

Learn About It MathSkills 121
 Joseph's favorite section of the Discover Science store displays dinosaurs on 3 shelves. Each shelf has 26 dinosaurs. How many dinosaurs are on the shelves?

Multiply. $3 \times 26 = n$

STEP 1 Estimate.
 3×26 is close to 90. 26 rounds to 30
 $3 \times 30 = 90$

STEP 2 Use base-ten blocks to show 3 groups of 26.

STEP 3 Multiply the ones.
 $3 \times 6 = 18$ ones
 Regroup 18 ones as 1 ten and 8 ones.

STEP 4 Multiply the tens.
 3×2 tens = 6 tens
 Add the 1 ten.
 6 tens + 1 ten = 7 tens

Solution: There are 78 dinosaurs on the shelves.
 Since 78 is close to 90, the answer is reasonable.

Student Book, grade 4, page 152

With respect to the learning of numbers, when students thoroughly understand concepts and procedures such as place value and operations with single-digit numbers they can extend these concepts to new areas (NRC, 2001, p. 119).

Another way to make connections is to use different representations for the same mathematical situation.

A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes (NRC, 2001, p. 119).

In grade 6, page 282, of *Houghton Mifflin Math*, students are shown two ways to represent the addition of integers: using counters and using a number line.

Different Ways to Add $+5 + -3$

Way 1 Use counters.

STEP 1 Show $+5$ and -3 with counters.
 $+5$: -3 :

STEP 2 Pair a yellow counter with a red counter until you have only one color left.
 $+5$: -3 : Each yellow-red pair represents $+1 + -1$, or 0.

STEP 3 Remove the zero pairs.
 $+5$: -3 : $\rightarrow = 0$
 There are 2 yellow counters left.
 So $+5 + -3 = +2$.

Way 2 Use a number line.

STEP 1 Begin at 0. Move right 5 units to show $+5$.

STEP 2 Then, starting at $+5$, move left 3 units to show -3 .

 The point at which you stop is the sum $+5 + -3$.
 $+5 + -3 = +2$

Student Book, grade 6, page 282