

Developing Mathematical Proficiency

Adding It Up: Helping Children Learn Mathematics, a report published by the National Research Council in 2001, is a summary of the work of a committee of members from diverse backgrounds who reviewed and synthesized relevant research on mathematics learning from pre-kindergarten through grade 8 (National Research Council [NRC], 2001, p. ix).

The committee chose the term *mathematical proficiency* to capture what it means for anyone to learn mathematics successfully. Mathematical proficiency is broken down into the following five components, or *strands*:

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one's own efficacy.

Interwoven Strands of Proficiency

Conceptual Understanding

Procedural Fluency

Strategic Competence

Adaptive Reasoning

Productive Disposition

(National Research Council, 2001)

The most important observation the committee makes and stresses is that **the five strands are interwoven and interdependent in the development of proficiency in mathematics** (NRC, 2001, p. 116).

Houghton Mifflin Math is based on the premise that the five strands of mathematical proficiency are interwoven. Pages 314 and 315 from grade 5 of this program can be used to illustrate this.

- The concept of multiplication of fractions is developed by connecting the fraction-bar model with the area model used for multiplication of whole numbers (*conceptual understanding*).
- The concrete model is used to develop the algorithm which is then practiced in the exercises (*procedural fluency*).
- Word problems involving multiplication of fractions are included in the lesson (*strategic competence*).

- Two problems subtitled Write About It and Explain require students to think logically, reflect, explain, and justify (*adaptive reasoning*).
- The word problems center around a school play, a topic that is age-appropriate (*productive disposition*).

This framework of mathematical proficiency, based on the five strands, is also similar to the framework used in recent mathematics assessments by the National Assessment of Educational Progress (NAEP), which features three mathematical abilities (conceptual understanding, procedural knowledge, and problem solving) and includes additional specifications for reasoning, connections, and communication (National Assessment Governing Board, 2000).

Lesson 2 Multiply Fractions
Objective: Find the product of two fractions.

Learn About It
To find a fraction of a fraction, you need to find the product of the fractions. You know how to use models to multiply fractions. Here are some other ways to find products of fractions.

Find $\frac{5}{6}$ of $\frac{3}{4}$.

Different Ways to Find $\frac{5}{6}$ of $\frac{3}{4}$

Way 1 You can multiply first, then simplify.
STEP 1 Multiply the numerators. Multiply the denominators.
 $\frac{5}{6} \times \frac{3}{4} = \frac{5 \times 3}{6 \times 4} = \frac{15}{24}$
STEP 2 Simplify the product if necessary.
 $\frac{15}{24} = \frac{5}{8}$

Way 2 You can simplify first, then multiply.
STEP 1 Rewrite using factors.
 $\frac{5}{6} \times \frac{3}{4} = \frac{5 \times 3}{2 \times 3 \times 4}$
STEP 2 Divide by common factors.
 $\frac{5 \times 3}{6 \times 4} = \frac{5 \times 3}{2 \times 8 \times 4}$
STEP 3 Multiply.
 $\frac{5 \times 1}{1 \times 2 \times 4} = \frac{5}{8}$

Solution: $\frac{5}{6}$ of $\frac{3}{4}$ is $\frac{5}{8}$.

Other Examples

A. Prime Factorization
 $\frac{9}{10} \times \frac{2}{3} = \frac{9 \times 2}{10 \times 3} = \frac{3 \times 3 \times 2}{2 \times 5 \times 3} = \frac{3 \times \cancel{2} \times \cancel{3}}{5 \times \cancel{3} \times 2} = \frac{3}{5}$
Notice that $\frac{2}{3}$ and $\frac{3}{3} = 1$. The product of 1 and any number is that number.

B. Fraction and Whole Number
 $\frac{1}{5} \times 4 = \frac{1 \times 4}{5 \times 1} = \frac{4}{5}$
 $\frac{1}{5} \times 4 = \frac{4}{5}$

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Guided Practice
Multiply. Write your answer in simplest form.
1. $\frac{2}{3} \times \frac{3}{5}$ 2. $\frac{3}{8} \times \frac{4}{5}$ 3. $\frac{5}{6} \times \frac{3}{10}$
4. $\frac{8}{4} \times \frac{3}{4}$ 5. $\frac{6}{5} \times \frac{2}{5}$ 6. $\frac{4}{7} \times 4$

Ask Yourself
• Can I use prime factorization?
• Did I write my answer in simplest form?

Explain Your Thinking
Why can you divide by common factors in the numerator and denominator to simplify a fraction?

Practice and Problem Solving
Multiply. Write your answer in simplest form.
7. $\frac{1}{2} \times \frac{5}{8}$ 8. $\frac{2}{5} \times \frac{5}{8}$ 9. $\frac{3}{6} \times \frac{5}{9}$ 10. $\frac{4}{6} \times \frac{5}{12}$ 11. $\frac{1}{6} \times \frac{2}{3}$
12. $\frac{1}{8} \times 6$ 13. $\frac{3}{8} \times 4$ 14. $9 \times \frac{5}{8}$ 15. $8 \times \frac{1}{6}$ 16. $2 \times \frac{3}{4}$

Algebra • Expressions Multiply. Write each answer in simplest form.
17. $\frac{a}{b} \times \frac{c}{d}$ 18. $2 \cdot \frac{z}{n}$ 19. $\frac{p}{q} \cdot \frac{r}{p}$
20. $3 \cdot \frac{6}{y}$ 21. $\frac{x}{8} \cdot \frac{2}{y}$ 22. $\frac{e}{x} \cdot \frac{b}{y}$

Solve.
23. Two fifths of a class is in the drama club. Only $\frac{1}{4}$ of them have parts in a play. What fraction of that class has parts in the play?
24. The middle school has 135 students. Two thirds of them are girls. How many of the students are girls? How many are boys?
25. Write About It How could you use two-color counters or coins to model $\frac{3}{4} \times \frac{1}{2}$?
26. Explain A bottle of water contains $\frac{7}{8}$ liter. Annette buys 12 bottles. How many liters does she buy? Explain.

Daily Review Find the product. (Ch. 3, Lesson 7)
27. 24×32 28. 18×43
29. 15×16 30. 92×87
31. 42×27 32. 88×13

Test Prep
33. John memorized $\frac{2}{5}$ of his lines for the school play. Then he memorized $\frac{1}{3}$ of what was left on Sunday. What fraction of his lines did he memorize on Sunday?
A $\frac{1}{9}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D 1

Extra Practice See page 301, Set A.

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