Developing the Prerequisites for Level 2 and Level 3 Methods of Adding and Subtracting

Kindergarten children are learning general level 1 numerical solution methods that they can extend to larger numbers. They are also working on all the prerequisites for the level 3 derived fact methods, such as make-a-ten (8 + 6 = 8 + 2 + 4 = 10 + 4 = 14).

- Seeing the tens in teen numbers (10 + 3 = 13).
- Knowing all the partners of 10 (e.g., 8 + 2 = 10).
- Knowing all the partners of numbers below 10 (e.g., know that 6 breaks into 2 and 4).


Learning Paths

In Unit 4, children continue to see the tens in teen numbers by making all of the teen numbers on the 1–20 Board with a 10-stick of 10 ones and extra ones cubes. This enables them to begin to see the pattern across all of the written teen numbers of the ones number in the right-hand place. Continued experience with the Number Tiles in which the ones number is placed on top of the 0 in the 10 tile, helps children see the teen number as 10 and 4; in this way the 1 in the tens place is understood as 10 (ten) instead of as 1. Work with equation cards that show 14 = 10 + 4 and with flashing fingers as ten fingers and 4 fingers relates the written numerals and objects and number words in meaningful ways.

Children work more intensively with partners of 10 in Unit 4 to build the second prerequisite listed above, and they continue to make partners of numbers 5 through 8 to build the third prerequisite.

Children also continue to build their understanding of adding and subtracting by telling and solving such problems with larger numbers in the class grocery store and by solving addition equations without a given scenario. They write the partner numbers within unknown boxes in an equation (e.g., 6 = [ ] + [ ]) and as an addition expression (e.g., 6 = _____ answered as 6 = 5 + 1).
In the English language, teen and 2-digit number words are complex and difficult to learn. By contrast, in some Asian languages the word for 13, for example, translates into “ten three.” In the English system, teen and 2-digit numbers look like two single-digit numbers written beside each other; nothing shows the ten value for the digit on the left. Young children need verbal and visual supports for understanding these number words and written numbers.

In this program, we provide this scaffolding by using tens and ones words, as well as standard number words, when working with teen and 2-digit numbers. We say 13 is thirteen and is 1 ten 3 ones. These words are used interchangeably and help reinforce the embedded ten-based thinking and place-value understanding. The words relate to the quantities children make or count that show the ten ones and some further ones. They also relate to the base-ten numerals in an equation such as $14 = 10 + 4$ using Number Tiles.

Other Useful References: Place Value


Getting Ready to Teach Unit 4

Using the Common Core Standards for Mathematical Practice

The Common Core State Standards for Mathematical Content indicate what concepts, skills, and types of problem solving children should learn. The Common Core State Standards for Mathematical Practice indicate how children should demonstrate understanding. These Mathematical Practices are embedded directly into the Student and Teacher Editions for each unit in Math Expressions. As you use the teaching suggestions, you will automatically implement a teaching style that encourages children to demonstrate a thorough understanding of concepts, skills, and problems. In this program, Math Talk suggestions are a vehicle used to encourage discussion that supports all eight Mathematical Practices. See examples in Mathematical Practice 6.

**COMMON CORE**

**Mathematical Practice 1**

*Make sense of problems and persevere in solving them.*

Children analyze and make conjectures about how to solve a problem. They plan, monitor, and check their solutions. They determine if their answers are reasonable and can justify their reasoning.

**TEACHER EDITION: Examples from Unit 4**

**MP.1 Make Sense of Problems** Check Answers

Elicit addition and subtraction story problems from children about the grocery store display. Then have the class find the total number of pieces of fruit. Have them check the answer by having the whole class count all of the fruit in the problem.

**MP.1 Make Sense of Problems** Act It Out

Ask a volunteer to act out someone buying vegetables at the classroom display. For example, Pete buys some vegetables. 6 of them are potatoes. The rest are carrots. How many carrots does Pete buy? Acting out story problems helps children think through the information with movement, props, and visuals.

Mathematical Practice 1 is integrated into Unit 4 in the following ways:

- Act It Out
- Analyze the Problem
- Make Sense of Problems
- Persevere in Solving Problems
**Mathematical Practice 2**  
**Reason abstractly and quantitatively.**

Children make sense of quantities and their relationships in problem situations. They can connect models and expressions to a given situation. Quantitative reasoning entails attending to the meaning of quantities. In this unit, this involves identifying a group with a given number of objects, counting out a group with a specified number of objects, connecting the addition and subtraction symbols with the addition and subtraction of quantities/numbers, recognizing the equality or inequality of amounts and correctly using the equal/not equal symbols, and determining the unknown quantity when the total and one partner are known.

**TEACHER EDITION: Examples from Unit 4**

**MP.2 Reason Abstractly** Have children practice identifying partners for 6 and 7, for example by playing the *Unknown Partner Game*. In this game children work in pairs and use centimeter cubes and Break-Apart Sticks to separate 6 or 7 into partners. Each set of partners is named.

**Lesson 2 ACTIVITY 1**

**MP.2 Reason Abstractly and Quantitatively** Connect Symbols and Models The addition symbol is shown by having children hold one arm vertically and one arm horizontally. Tell children that for the addition symbol things are put together, so they show it by putting arms together. The subtraction symbol is shown by having children hold one arm horizontally (like the subtraction symbol). Children then open and close their hand as if they were grasping things and move their arm to the side to pull away the things they are taking. Altogether, this action links the visual appearance of the subtraction symbol with a meaning of subtraction as “taking away.”

**Lesson 12 ACTIVITY 3**

**Mathematical Practice 2** is integrated into Unit 4 in the following ways:
- Connect Symbols and Models
- Connect Symbols and Words
- Reason Abstractly
- Reason Abstractly and Quantitatively
- Reason Quantitatively
Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Children use stated assumptions, definitions, and previously established results in constructing arguments. They are able to analyze situations and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

Children are also able to distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Children can listen to the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Talk is a conversation tool by which children formulate ideas and analyze responses and engage in discourse. See also MP.6 Attend to Precision.

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**What's the Error**

**MP.3 Construct a Viable Argument**

How can we make sure that we find all of the partners of ten?

- **Jill:** We can use Square-Inch Tiles to show the partners.
- **Eduardo:** We have to remember to turn over one more tile each time.
- **Bonnie:** We make the first partner one more each time.
- **Jill:** We can check the partners we find. We can see if all of the numbers are there.
- **Eduardo:** The numbers should be 1, 2, 3, 4, 5, 6, 7, 8, and 9.
- **Bonnie:** If we have all of those numbers, we have found all of the partners of ten.

You are correct.

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**Mathematical Practice 3**

is integrated into Unit 4 in the following ways:

- Compare Methods
- Compare Representations
- Construct a Viable Argument
- Justify Conclusions
- Puzzled Penguin
**Mathematical Practice 4**

Model with mathematics.

Children can apply the mathematics they know to solve problems that arise in everyday life. This might be as simple as writing an equation to solve a problem. Children might draw pictures to lead them to a solution for a problem.

Children apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation. They are able to identify important quantities in a practical situation and represent their relationships using such tools as sketches and tables.

**TEACHER EDITION: Examples from Unit 4**

**MP.4 Model with Mathematics** Ask children to count out 10 Square-Inch Tiles of the same color. Direct children to arrange their Square-Inch Tiles in a horizontal row of 10.

- Now make your bugs go to sleep by placing all of them dot side down, so you can’t see them. How many are sleeping? **all 10 of them**
- Have children wake one bug.
- One bug wants to get up and play. Let’s turn one over. How many bugs are sleeping now? **9**
- How many bugs are awake? **1**
- How many bugs in total? **10**
- Put your Break-Apart Stick between the sleeping bugs and the bug that is awake, so you can see them clearly.

**MP.4 Model with Mathematics** Write an Expression The emphasis in this unit is on addition and subtraction problems within 10. Keep encouraging children to make up problems with these numbers. Use the grocery store display you made from the fruit that children colored during Unit 4, Lesson 1. Ask for a volunteer to go to the grocery store and buy two kinds of fruit to make a fruit salad. The child will decide how many pieces of each kind of fruit to buy.

- Maria, what two kinds of fruit do you want to buy? **I want to buy oranges and bananas.**
- How many oranges and how many bananas will you buy? **3 oranges and 4 bananas**

Have Maria take the fruit from the store while the whole class counts with you, the storekeeper, to check that the correct amount of fruit is being bought. Record Maria’s fruit salad equation on the board: $3 + 4 = \_\_\_\_\_\_\_\_\_.

**Mathematical Practice 4** is integrated into Unit 4 in the following ways:

- Model with Mathematics
- Write an Expression
Mathematical Practice 5

Use appropriate tools strategically.

Children consider the available tools and models when solving mathematical problems. Children make sound decisions about when each of these tools might be helpful. These tools might include paper and pencil for drawings and computation, manipulatives, or even their fingers. They recognize both the insight to be gained from using the tool and the tool's limitations. When making mathematical models, they are able to identify quantities in a practical situation and represent relationships using modeling tools, such as tables, expressions, and equations.

Modeling numbers in problems and in computations is a central focus in Math Expressions lessons. Children learn and develop models to solve numerical problems and to model problem situations. Children continually use both kinds of modeling throughout the program.

MP.5 Use Appropriate Tools

Use a Concrete Model

Children make a Teen Number Book in Unit 4 to help them visualize teen numbers as a group of ten ones and some extra ones. It also helps to reinforce this concept as children show and write teen numbers. After the book has been assembled, allow children to explore various ways to use it. Show children how to turn the pages to say the numbers in order from 11 to 20. Ask questions such as the following.

- Point to the page that shows the partners for 13.
- What is an equation for 17?
- Find the number that shows ten ones and 5 ones.

Model the Math

Children make numbers 11 to 20 using 10-sticks and centimeter cubes on the 1–20 Board. They make 11 with one 10-stick and one centimeter cube. They learn how to make 12 with one 10-stick and 2 centimeter cubes. They continue to model the numbers 13–19 on their 1–20 Boards.

- What are three different ways you can make 20? use 20 centimeter cubes, one 10-stick and 10 centimeter cubes, or two 10-sticks

Mathematical Practice 5 is integrated into Unit 4 in the following ways:

- Use Appropriate Tools
- Use a Concrete Model
- Model the Math
Mathematical Practice 6
Attend to precision.

Children try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. When restating story problems, they are careful about specifying units to clarify the correspondence with quantities in the problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. Children give carefully formulated explanations to each other.

TEACHER EDITION: Examples from Unit 4

MP.6 Attend to Precision Explain a Representation Give each group of children a variety of cylinders, including different sized cylinder-shaped objects.
• Who knows the name of these shapes? cylinders
• What do you see that is the same about all of the cylinders? They can roll. They have a circle on both ends. They all look the same but the sizes are different.
• What have you seen that is shaped like a cylinder? a roll of paper towels, a can, a roll of wrapping, a glass
• Are cylinders solid shapes? How do you know? They are solid shapes because they they go across, backward and forward, and up and down.

MP.6 Attend to Precision Describe a Method Ask two children to describe how they remember the addition sign.
• How many parts, or lines, does the addition sign have? The addition sign has two parts.
• What does the addition sign show you to do? It shows putting together two partners.
• How do the two parts of the addition sign help you remember to add two things? The addition sign is made up of two things, and we add two things.
• What do you do when you see an addition sign with two partners and an equal sign? You see the partners and find the number those partners make.

Mathematical Practice 6 is integrated into Unit 4 in the following ways:
Attend to Precision
Describe
Describe a Method

Explain a Representation
Explain a Solution
**COMMON CORE**

**Mathematical Practice 7**

**Look for structure.**

Children analyze problems to discern the structure. They draw conclusions about the structure or the relationships they have identified.

**TEACHER EDITION: Examples from Unit 4**

**MP.7 Look for Structure**  Talk about the partners in each Math Mountain. Continue working with those who need support. When children finish, ask them to draw a big Math Mountain on a sheet of paper. Let children draw Tiny Tumblers the way they picture them rolling and playing on the mountains. You can extend the discussion by asking the class what they think the Tiny Tumblers play, how they have fun, what their cozy living places look like, and so on. Stimulate children’s thinking by asking questions.

**Mathematical Practice 7** is integrated into Unit 4 in the following ways:

- **Look for Structure**
- **Identify Relationships**

**Lesson 11**

**MP.7 Look for Structure**  Identify Relationships  Have the class decide what they want the Square-Inch Tiles to be. In this example, the dot side of the tile is a little bug. Sometimes the bugs are awake and turned up so that they can be seen, and sometimes they are sleeping under the tile and turned over so that they are hidden. There are always 10 bugs in total. Continue in this manner, having children turn over the bugs one at a time. Record the partners on the board. Help children notice that they can quickly see how many bugs are asleep and awake when they are arranged in two rows of 5. Remind children that using two rows of 5 will help them count faster. Have children discuss the numbers and sequences they see in the chart on the board.
**COMMON CORE**

**Mathematical Practice 8**

*Look for and express regularity in repeated reasoning.*

Children use repeated reasoning as they analyze patterns, relationships, and calculations to generalize methods, rules, and shortcuts. As they work to solve a problem, children maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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**TEACHER EDITION: Examples from Unit 4**

### MP.8 Use Repeated Reasoning

**Generalize**

Review making teen numbers with 10-sticks and centimeter cubes and have children make observations. For example, children may notice that each of the teen numbers has one 10-stick. They may see that each number has one more cube than the number before it. Children might also discern that the number in the right column matches the number of centimeter cubes and the number in the left column matches the number of 10-sticks.

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### MP.8 Use Repeated Reasoning

**Generalize**

Distribute the Teen Equation Cards. You may want to have some children work only with Set A of the cards. Children will place the cards on the corresponding teen numbers on the 1–20 Board. Cards can be placed below the teen numbers at the top of the 1–20 Board or at the bottom, covering the matching teen equations.

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**Mathematical Practice 8** is integrated into Unit 4 in the following ways:

- **Draw Conclusions**
- **Generalize**
- **Use Repeated Reasoning**

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**FOCUS on Mathematical Practices**

Unit 4 includes a special lesson that involves solving real world problems and incorporates all 8 Mathematical Practices. In this lesson children use what they know about the attributes of three-dimensional shapes to compare the shapes.

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**STUDENT EDITION: LESSON 22, PAGES 199–200**

1. Circle the objects shaped like cubes on the top shelf. Circle the objects shaped like cylinders on the middle shelf. Circle the objects shaped like spheres on the bottom shelf.

2. Color each kind of shape.
Getting Ready to Teach Unit 4

Learning Path in the Common Core Standards
In this unit, children continue the study of the teen numbers and their structure as ten ones and more ones. The focus on working with and learning the partners of numbers from 2–10 helps children prepare to write and use equations to represent partner situations. They work within the context of a grocery store scene to make up and solve story problems and to sort and compare objects. In geometry, they learn to use attributes to identify three-dimensional shapes.

Help Students Avoid Common Errors
Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that children make. They enjoy teaching Puzzled Penguin the correct way, telling why this way is correct, and explaining why the error is wrong. The common errors are presented as letters from Puzzled Penguin to the children:

- **Lesson 1**: counting 14 instead of 15 cherries
- **Lesson 8**: incorrectly identifies partners for 10, writing 10 = 6 + 5 and 10 = 3 + 6
- **Lesson 17**: after subtracting, finds 3 as the answer for 4 + 1 instead of finding 4 + 1 = 5
- **Lesson 19**: incorrectly writing an equation for the partners of 6 as 6 = 4 – 2 instead of 6 = 4 + 2

In addition to Puzzled Penguin, other suggestions are listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.
Story Problems Children make up addition and subtraction story problems about buying and selling fruits and vegetables in a grocery store. These story problems should involve totals from 6–10. In Lesson 15, children make up stories for a context of their choice.

Represent the Situation Beginning in Lesson 6, children make simple math drawings to show an addition or subtraction situation and write the expression (or equation) represented by the drawing. As noted in the lessons, emphasize that drawings should be of simple objects that are easy to draw, such as circles, lines, or boxes.

As you record work with addition and subtraction situations, be sure that you write the equation (for example, \(6 + 2 = 8\) or \(7 - 3 = 4\)), but do not expect that many children will be able to do this. During the research phase of this program, it was observed that writing a full equation is difficult for most kindergarteners when they begin using addition or subtraction to solve a problem, something that you have also probably observed. However, most children will be able to write the expression that describes the drawing and/or situation (for example, \(6 + 2\) or \(7 - 3\)).

Support Subtraction Understanding Call attention to the connections between the partners of a number and the known and unknown addends in subtraction. For example, in \(7 - 3 = 4\), the partners of 7 are 3 and 4.

Model subtraction in your drawings by drawing a long line through the objects taken away and point out that the line looks like a big subtraction sign. Children may use other methods that make sense to them, such as drawing a shorter line or an X through each object. Making a point of drawing the line through the first objects in the drawing prepares children for subtracting by counting on in Grade 1.
Partners of 10

**With a Break-Apart Stick** Finding partners of 10 prepares children for the Make a Ten subtraction strategy they will learn in Grade 1. Children use the Break-Apart Stick to find partners of 10. In a fun context of sleeping and waking bugs, children use the stick to break apart a 10 they have made with Square-Inch Tiles, either in 1 row of 10 or in 2 rows of 5. This activity helps children solidify subitizing skills. You record results on the board. Notice that the equation form you will use emphasizes that a number is broken apart into partners.

![Break-Apart Stick diagram]

**With a Line** Children move from using the manipulative with objects to finding partners of pictured objects. Instead of a Break-Apart Stick, they draw a line to separate a row of 10 objects into partners and then complete a partner equation for the partners they made.

![Line diagram]

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**From THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON COUNTING AND CARDINALITY**

From subitizing to single-digit arithmetic fluency Perceptual subitizing develops into conceptual subitizing—recognizing that a collection of objects is composed of two subcollections and quickly combining their cardinalities to find the cardinality of the collection (e.g., seeing a set as two subsets of cardinality 2 and saying “four”).
Teen Numbers as 10 Ones and Extra Ones

Model Ten Ones and Extra Ones  The models for this unit’s activities with teen numbers are 10-sticks and centimeter cubes. Children model the teen numbers using the 10-stick to represent the 10 ones part of the teen number and the cubes to represent the extra ones. As they model numbers on the 1–20 Board, they see how the numbers change as another one is added. This model helps children build the concept that the 1 in a teen number is not one, but is 10 ones. This fundamental concept gives the children a strong foundation on which they can build their place value concepts in later grades.

Teen Equation Cards  Children also use the 1–20 Board with the Teen Equation Cards. They match the cards to the columns on the board. The two sets of cards reinforce visually the relationship between the partners of a number and the addends that make that number as a total. So, for example, the children begin to see that $11 = 10 + 1$ and $10 + 1 = 11$.
The idea that the total can be on either the right or the left side of an equation supports a concept of equality that is important for children’s understanding of algebra.

**Teen Number Book** As a summarizing activity for the work with teen numbers, children will draw models for the teen numbers on separate pages in the Teen Number Book. After children complete pages for all the teen numbers, they will make covers for the books and assemble them. You can then staple the pages together for each child. Think of ways to use the Teen Number Book in activities for a few days so children know that their work produced something useful. Then have children take the books home to share with their families.

*from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING*

**Work with numbers from 11 to 19 to gain foundations for place value** The numerals 11, 12, 13, ..., 19 need special attention for children to understand them. The first nine numerals 1, 2, 3, ..., 9 and 0 are essentially arbitrary marks. These same marks are used again to represent larger numbers. Children need to learn the differences in the ways these marks are used. For example, initially, a numeral such as 16 looks like “one, six,” not “1 ten and 6 ones.”
Partners of Numbers 2–10 and Equations

Knowing partners of numbers through 5 helps children learn to add and subtract within 5. Working with partners of teen numbers and numbers through 10 helps children build subitizing skills and begin to represent addition and subtraction situations with expressions and equations. In this unit, children continue activities from earlier units and build on what they know to carry out new activities.

Math Mountains  A new model for addition and subtraction situations is introduced in Lesson 8. The Math Mountain is a powerful visual representation that relates addition and subtraction and will be used in later grades as well as in kindergarten. For kindergarten children, the model begins with an imaginative context, Tiny Tumblers who live on Math Mountains. The number of Tiny Tumblers who live on a Math Mountain is the same as the number at the top of the Math Mountain. The numbers of Tiny Tumblers who play on the two sides of a Math Mountain must add to the number at the top of the mountain, but different numbers of tumblers may be on the sides.

Children will draw the Tiny Tumblers as circles on the sides of Math Mountains on their Student Activity Book pages. The Tiny Tumblers on each side of a Math Mountain represent the partners of the number at the top of the Math Mountain.

Equations  Children build algebraic concepts as they work with partners, both of numbers 2–10 and teen numbers. They complete equations for break-apart activities, first by writing the partners in answer boxes, next by writing the total in an answer box, and finally by writing a full equation for a partner situation.
Sorting, Comparing, and Ordering

Sorting and Comparing Objects  Children sort the fruits and vegetables from the Grocery Store scenario by attributes. They first compare by counting two groups of objects to determine which group is greater or less. They also learn to compare amounts by drawing lines to match objects. They then record the results of the matching with a G for “greater than” or an L for “less than.”

Dot-to-Dot Picture  Finding numbers in order to complete a dot-to-dot picture can be an enjoyable way for children to demonstrate their proficiency with the count sequence and for you to informally assess this skill. You may want to write the name of the object on the board when most children have finished so that children who can print may label their pictures.
Attributes of Three-Dimensional Shapes

Among the Geometry skills in the Kindergarten Common Core State Standards are identifying and describing shapes, correctly naming shapes, and describing relative positions of objects. The lessons in this unit focus on these skills as they relate to three-dimensional objects, including cubes, cylinders, and cones.

Path to Fluency

The Common Core State Standards require that kindergarten children fluently add and subtract within 5. One way to acquire this fluency is with persistent practice. The Path to Fluency exercise sets on Student Activity Book pages in these lessons provide practice with addition and subtraction within 5 to help children achieve this goal.

\[
\begin{align*}
3 - 2 &= 1 \\
5 - 5 &= 0 \\
2 - 2 &= 0 \\
4 - 1 &= 3 \\
4 - 3 &= 1 \\
3 - 1 &= 2
\end{align*}
\]

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Working within 10 Later in the year, students solve addition and subtraction equations for numbers within 5, for example, \(2 + 1 = \square\) or \(3 - 1 = \square\), while still connecting these equations to situations verbally or with drawings. Experience with decompositions of numbers and with Add To and Take From situations enables students to begin to fluently add and subtract within 5.