MATH Expressions Common Core

BUILDING A New Standard OF Success

A Math Talk Community in *Math Expressions Common Core*

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WHAT IS A MATH TALK COMMUNITY?

The Common Core State Standards (CCSS) expect students to reason about mathematical ideas and to discuss and explain that reasoning. Visual representations (e.g., diagrams) are used to support understanding. Steps in written methods can be supported by steps in visual representations that students draw. The eight mathematical practices in the CCSS also involve reasoning and explaining and using drawings.

Math Expressions supported these CCSS emphases before the CCSS even existed because these emphases came from research used in designing ***Math Expressions*** and the CCSS. For example, the National Council for Teachers of Mathematics (NCTM) Standards and two National Research Council reports (*Adding It Up: Helping Children Learn Mathematics* and *How Students Learn: Mathematics in the Classroom*) emphasize the need for students to discuss their mathematical thinking as a way to increase understanding. The *CMW Research Project* that developed the ***Math Expressions*** program carried out research to identify crucial aspects of such discussions, and to identify levels through which teachers could transition from traditional teacher-focused instruction to productive student-to-student discussion, monitored and supported by the teacher, as recommended by the National Research Council reports. We also sought in this project to identify other features that could support effective discussions.

We found that the term Math Talk was effective in communicating with teachers and students to achieve the focus on discussion we desired. Our vision was for Math Talk to be an instructional conversation directed by the teacher, but with as much student-to-student talk as possible. Math Talk is focused on developing the understanding of all students in the class and on moving everyone along the learning path for the topic.

Math Talk needs to occur in a safe and supportive classroom environment if learning is to occur. We describe such an environment as a nurturing, meaning-making Math Talk Community focused on teaching/learning by the students and the teacher. The rest of this paper describes this community in more detail.

In true Math Talk, teachers create a Math Talk Inquiry environment and encourage constructive discussion of problem-solving methods through well-defined classroom activity structures, based on the four components of a Math Talk Learning Community. Table 2 (see next page) describes the four components of Math Talk (questioning, explaining math thinking, source of math ideas, and responsibility for learning), and summarizes roles of the teacher and of the students at the highest stage, Stage 3. There are also many Math Talk boxes in the ***Math Expressions*** Teacher Editions that provide specific help with Math Talk.

THE SOLVE AND DISCUSS STRUCTURE

We found that two kinds of Solve, Explain, Question, and Justify (often shortened in use to Solve and Discuss) classroom activity structures were very effective in engaging all students in Math Talk (see Table 1). In both structures, all students solve problems simultaneously. In the first structure, as many students as possible go to the board to solve a problem while the rest of the students work at their seats. Then the teacher selects two or three students from the board who have interesting solutions, or need the chance to explain their work, to talk about their solution. Only two or three students need to explain their work because students usually cannot maintain concentration for more than two or three discussions of the same problem. Next, a different group of students goes to the board to solve the next problem. This process is very motivating. Most students enjoy solving problems at the board even if they do not get the chance to explain their work. While the students are working at the board, the teacher has a chance to see how solutions evolve. The teacher also gets a good sense for how individual students are doing. In one class period, many or even all of the students can get a turn at the board.

The second effective classroom structure allows every student in the classroom to explain his or her solution. Every student solves a problem at his or her seat. Then two or three students are selected by the teacher to go to the board to draw their solutions. The students left at their desks then pair up and explain their solutions to each other. Then the class discusses the solutions of those students at the board. Students at their seats can write their solutions on paper, which can be picked up and skimmed later by the teacher to see how students are progressing. Another option is to have students at their desks solve problems on the large (12" by 19"), individual ***Math Expressions*** dry-erase boards called MathBoards. These MathBoards permit the teacher to send any additional student to the board for another explanation of the discussed problem, because the drawings on the MathBoard are large enough to be seen by classmates.

An important feature of both of these classroom structures is that no class learning time is lost. In other approaches, when students are sent to the board to draw their work, the rest of the class remains at their seats doing nothing. In the case of these ***Math Expressions*** structures, the students at their desks are just as involved in the problem solving as those at the board. Sometimes a step-by-step variation of these activity structures is helpful. Teachers can have each student explain one step of a solution at a time, until a final solution is reached. Another method is to put students in pairs. The pairs can solve together and explain their work, with the less advanced student explaining first, and the other, more advanced student expanding and clarifying as needed.

Solve and Discuss	
1	**Solve:** All students solve.
2	**Explain:** One student explains and then asks, "Are there any questions?"
3	**Question:** Other students ask questions to clarify or extend.
4	**Justify:** The original explainer responds to the questions by explaining more (justifying the original explanation).
Any student at any time can ask for help from anyone. Typically another student explains, so the class loops through 2, 3, and 4 again. The discussion can now also contrast and compare the first and second solutions as well as others in the past.	

Table 1

Components of the Math Talk Learning Community			
A. Questioning	**B. Explaining math thinking**	**C. Source of math ideas**	**D. Responsibility for learning**
Overview of shift over Stages 0–3: The classroom community grows to support students acting in central or leading roles, and shifts from a focus on answers to a focus on mathematical thinking.			
There is a shift from the teacher as questioner to the students and teacher as questioners.	The students increasingly explain and articulate their math ideas.	There is a shift from the teacher as the source of all math ideas to students' ideas also influencing the direction of lessons.	The students increasingly take responsibility for learning and evaluation of others and of themselves. Math sense becomes the criterion for evaluation.
Stages in the Math Talk Learning Community			
Stage 0: This is a traditional teacher-directed classroom with brief answer responses from students.			
Stage 1: The teacher is beginning to pursue student mathematical thinking. The teacher plays a central role in the Math Talk community.			
Stage 2: The teacher models and helps students build new roles. Some co-teaching and co-learning begin as student-to-student talk increases. The teacher physically moves to the side or back of the room and directs from there.			
Stage 3: The teacher is a co-teacher and co-learner. The teacher monitors all that occurs and is still fully engaged. The teacher is ready to assist, but now in a more peripheral and monitoring role (coach and assister).			
The teacher expects students to ask one another questions about their work. The teacher's questions still may guide the discourse. *Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions of each other and listen to responses. Many questions are "why?" questions that require justification from the person answering. Students repeat their own or other students' questions until they are satisfied with the answers.*	*The teacher follows along closely to student descriptions of their thinking, encouraging students to make their explanations more complete; he or she may ask probing questions to make explanations more complete. The teacher stimulates students to think more deeply about strategies.* *The students describe more complete strategies; they defend and justify their answers with little prompting from the teacher. The students realize that other students will ask them questions, so they are motivated and careful to be thorough. Other students provide support with active listening.*	*The teacher allows for contributions from students during his or her explanations; he or she lets the students explain and "own" new strategies. The teacher is still engaged and deciding what is important to continue exploring. The teacher uses student ideas and methods as the basis for lessons or mini-extensions.* *The students contribute their ideas as the teacher or other students are teaching, confident that their ideas are valued. The students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons.*	*The teacher expects students to be responsible for co-evaluation of everyone's work and thinking. He or she supports students as they help one another sort out misconceptions. He or she helps and/or follows up when needed.* *The students listen to understand, then initiate clarifying other students' work and ideas for themselves and for others during whole-class discussions, as well as in small group and pair work. The students assist each other in understanding and correcting errors.*

Table 2

This table comes from the research paper "Describing levels and components of a math-talk community" by K. Hufferd-Ackles, K. C. Fuson, and M. G. Sherin. 2004. *Journal for Research in Mathematics Education* 35 (2): 81–116. A later version of this paper revised for teachers emphasizes the importance of math representations in math talk (see the final page).

Responsive Means of Assisting Learning by Teacher and Students		
Engage and Involve		
Manage		
Coach	→	Show/explain • Model (as a student) • Instruct/explain (as a teacher) Focus and extend • Cognitive structure and question • Question • Give feedback

Table 3

EVERYONE ASSISTS LEARNING

Table 3 (above) shows ways in which everyone in the class can assist the learning of others. Engaging and involving includes inviting all students to share ideas and questions, promoting analysis and discussion, and expecting that all students participate in developing understanding together in the community. Managing includes helping students monitor, be responsible for, and take ownership of their own learning. Initially the teacher engages and involves and also manages, but with encouragement and support students can also assist the Math Talk Community in these ways.

Coaching involves two major categories. In show/explain classroom members may model as a student or instruct/explain as a teacher. In focus and extend, the teacher or student coach may cognitively structure and clarify, question, or give feedback. These specific means of assisting learning occur as students engage in the CCSS mathematical practices shown in Table 4. They lead attention that enables everyone to connect visual, verbal, and gestural aspects of teaching/learning.

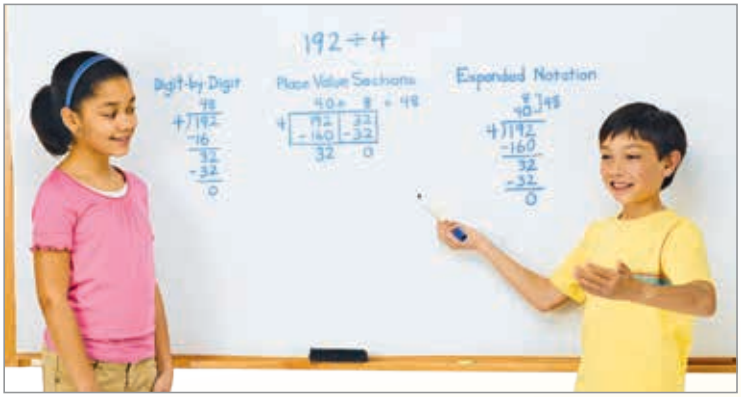
Common Core Mathematical Practices Used in a Math Talk Community	
Math Sense-Making 1 Make sense of problems and persevere in solving them. 6 Attend to precision.	**Math Drawings** 4 Model with mathematics. 5 Use appropriate tools strategically.
Math Structure 7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.	**Math Explaining** 2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others.
Teachers continually assist students to do math sense-making about math structure using math drawings to support math explaining.	

Table 4

LEARNING MATH TALK

Effective Math Talk cannot be implemented in a classroom overnight. A teacher must work his or her students up to Stage 3 Math Talk over time. It often takes two or three months to build a classroom up to Stage 3 if students are not familiar with Math Talk from the start. Initially, the teacher and more advanced students will do a lot of modeling of solving and explaining for other students. In the beginning, teachers also concentrate on building listening skills by asking students to repeat a problem, question, or explanation in their own words. Teachers also elicit questions from students. Questions may apply to any topic (e.g., *How does your method relate to the method that Sam just explained?* or *Why did you use this method?)* or may apply specifically to a given math topic. In the latter case, the teacher needs to model some of these questions to use new math vocabulary, though often more advanced students can also think of such questions. Students need to learn to stand beside their drawings and numerical work, and point to parts of it with a pointer as they explain. Students often initially explain only one part of their thinking, or explain it incompletely. Using questions can help expand a student's explanation. The teacher or another student may also expand or clarify the student's explanation through questions, while always checking with the original explainer to be sure that the added information is what the student intended to convey. In addition to helping students learn Math Talk methods, teachers often need to adjust to the Math Talk structures themselves. They must learn to wait patiently and use a "bite your tongue" strategy to allow student talk to emerge. They must also physically move to the side or back of the room to facilitate student-to-student talk so that the explainer looks at classmates and not at the teacher. From the side or back of the room, a small gesture can be used to remind the explainer to look at classmates rather than at the teacher. Teachers can provide community assistance by asking explainers if they need help, but they also need to allow wait time before doing so. Shy students initially may need the presence of a friend at the board with them, even if the friend does not help with the explaining. As teachers provide the space and support for students' voices to emerge, they often report being amazed by the mathematical thinking their students are able to express.

Talking must be safe for teachers and students. Initially the teacher needs to emphasize that Math Talk is not a test. It is helping everyone learn more by sharing your thinking. It is vital that the teacher emphasize: No making fun of anyone ever. The teacher does not have to know the answer to every question immediately. If no one including the teacher can answer a question or be clear about something, model extended problem solving by saying: "This is tricky. I need to think more about that. Let's all think about it and talk about it tomorrow." And then talking to other teachers is a good way to get help for a tricky issue.



A NURTURING, MEANING-MAKING MATH TALK COMMUNITY

The Math Talk environment is a helping community where inquiry and meaning-making are nurtured, and everyone is a teacher and a learner. This creates a secure base for learning and for Math Talk. It enhances everyone's mathematical understanding, competence, and confidence. Teachers build the Helping Community daily by helping students

learn how to help each other at the board, in pairs or groups, or when working individually. ***Math Expressions*** was developed through ten years of intensive classroom research, in many classrooms, with students from many different cultural and linguistic backgrounds. The Nurturing, Meaning-Making Math Talk Helping Community enables students from all backgrounds to bring their family and cultural experiences into the classroom and be validated, affirmed, and understood.

The Math Talk approach used by ***Math Expressions*** supports deeper understanding and more complex language learning than other reform approaches, because the ***Math Expressions*** program provides research-based learning paths that move all students forward. Students develop the prerequisite understandings so that they can invent interesting methods. Research-based, accessible strategies are taught so that everyone has an effective method. These relate to common methods so that all methods can be discussed and related. Many students learn, relate, and explain multiple methods. Math drawings enable students to solve and explain more effectively and enable listeners to understand and question more effectively. This process of discussing the whole developmental range of solution methods permits differentiated instruction to occur in whole-class activities, but also enables students to move forward to a mathematically desirable and effective method. ***Math Expressions*** truly supports teachers as they develop a Nurturing, Meaning-Making Math Talk Inquiry Classroom.

The opportunity for all students to explain their math thinking over time is especially valuable for students learning English, as well as for native speakers advancing their verbal communication skills. Ultimately, developing understanding and verbal communication will aid all students in their future education and careers. In addition to verbal communication, the use of math drawings is central to Math Talk. Math drawings can show the quantities in a computation and relate them to a written numerical method, or can show the situation in a word problem. The math drawings help everyone understand the student's math thinking. The special learning supports on the MathBoards enable students to learn meaningful drawings rapidly, and then the open space on the MathBoards is used for math drawings.

QUESTIONS

Questions from the teacher and from students play a crucial role in the Math Talk Community. These questions can be genuine questions, or they can be "teacher-y" questions asked to extend the learning of others. Many students love to develop and ask such questions. Questions can be general or can apply to specific math concepts. Some questions are shown in Table 5. The teacher initially may model many of these questions, but with support and encouragement many students can ask such questions independently. This increases the depth of functioning of the Math Talk Community. Posters of questions can be made and hung on the wall. A blank poster put up for each new unit can encourage students to think of and add questions that focus specifically on that unit.

Questions for Teacher and Students

Elicit student thinking

- So, what is this problem about?
- Tell us what you see.
- Tell us your thinking.

Support student thinking

- What did you mean when you said _____?
- What were you thinking when you decided to _____?
- Show us on your drawing what you mean.
- Use wait time: Take your time.... We'll wait....

Extend student thinking

- Revoicing: So you're saying that _____?
- Now that you have solved the problem in that way, can you think of another way to work on this problem?
- How is your way of solving like _____'s way?
- How is your way of solving different from _____'s way?
- What would happen if _____?
- How can we check to be sure that this is a correct answer?
- Is that true for all cases?
- What pattern (structure) do you see here?

Increase participation of other students in the conversation

- Prompt students for further participation: Would someone like to add on?
- Ask students to restate someone else's reasoning: Can you repeat what _____ just said in your own words?
- Ask students to apply their own reasoning to someone else's reasoning:
- Do you agree or disagree, and why?
- Did anyone think of this problem in a different way?
- Does anyone have the same answer, but got it in a different way?
- Does anyone have a different answer? Will you explain your solution to us?

Probe specific math topics

- Why did you make a new hundred?
- Where did you write your new one ten? Where is your new one ten on your drawing?
- Why did you write the product of 6 x 4 in the hundreds place? And show us that part on your area model.
- Why did you choose 12 to be the new unit fraction to add 3/4 and 5/6?

Table 5

CLOSING COMMENTS

Having students make math drawings to support and show their thinking has been emphasized throughout. These drawings also support student explaining and help classmates and the teacher understand the explainer's thinking. Making math drawings was emphasized in a shortened and updated adaptation of the 2004 paper by having a column "mathematical representations" in the updated table (Hufferd-Ackles, Fuson, & Sherin, 2015). Students follow a progression from learning to create math drawings to depict their math thinking to labeling their math drawings so others are able to follow their math thinking to suggesting edits in others' math drawings.

The term Math Talk Learning Community used in the 2004 paper was used to emphasize that each member of the learning community, including the teacher, learned mathematics when math talk was nurtured and developed. This community may be a bit different each year because each group of students may develop a math conversation culture specific to that group. Particular questions will become part of a given classroom culture and can be asked across topics. Teachers can support students by making their own question table as they get started and find questions that work well with their classroom community. Then they can post a student version in the classroom as a reference.

You can watch a webcast about developing a math-talk community and watch such classrooms in action at these web locations:

Math Talk Community Overview (Part 1):
brainshark.com/hmhsupp/vu?pi=zIFzajQvYz6k6sz0&intk=216677776

Getting Started with Math Talk (Part 2):
brainshark.com/hmhsupp/vu?pi=zGhzYG1dXz6k6sz0&intk=447098499

REFERENCES

Fuson, K. C., Y. De La Cruz, S. Smith, A. Lo Cicero, K. Hudson, P. Ron, and R. Steeby. 2000. "Blending the best of the 20th century to achieve a Mathematics Equity Pedagogy in the 21st century." In M. J. Burke and F. R. Curcio (Eds.), *Learning Mathematics for a New Century.* pp. 197–212. Reston, VA: NCTM.

Fuson, K. C. 2003. "Developing mathematical power in whole number operations." In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics.* pp. 68–94. Reston, VA: NCTM.

Fuson, K. C., M. Kalchman, and J. D. Bransford. 2005. "Mathematical understanding: An introduction." In M. S. Donovan & J. D. Bransford (Eds.), *How Students Learn: Mathematics in the Classroom.* pp. 217–256. Washington, DC: National Academy Press.

Fuson, K. C., and A. Murata. 2007. "Integrating NRC principles and the NCTM Process Standards to form a Class Learning Path Model that individualizes within whole-class activities." *NCSM Journal of Mathematics Education Leadership* 10 (1), 72–91.

Hufferd-Ackles, K., K. C. Fuson, and M. G. Sherin. 2004. "Describing levels and components of a math-talk community." *Journal for Research in Mathematics Education* 35 (2): 81–116.

Hufferd-Ackles, K., K. C. Fuson, and M. G. Sherin. 2015. Describing levels and components of a Math-Talk Learning Community. In E. A. Silver and P. A. Kenney (Eds.), *More lessons learned from research: Volume 1: Useful and usable research related to core mathematical practices,* (pp. 125-134). Reston, VA: NCTM. [Read the paper at www.sesp.northwestern.edu/profile/?p=61&/KarenFuson/]

Kilpatrick, J., J. Swafford, and B. Findell (Eds.). 2001. "Adding It Up: Helping Children Learn Mathematics." Washington, DC: National Academy Press.

National Council of Teachers of Mathematics. 2000. "Principles and Standards for School Mathematics." Reston, VA: NCTM.

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