

Math Background

Multiplicative Contexts for Understanding Proportions

The research literature contains many studies documenting the errors students make in solving proportion problems and how difficult this topic remains even through high school. The most typical error is to solve proportion problems additively instead of seeing the multiplicative relationships involved. Students also often only see the 4 numbers involved in the proportion and do not see how these numbers are just examples taken from a whole table of multiplications of the ratios involved. Also, proportions are often solved by using fractions, and some approaches even define proportions as fractions. But ratios and proportions have properties that differ from fractions (for example, you can add ratios to make new ratios, but you cannot add fractions with different denominators to make new fractions).

For these reasons the *Math Expressions* research-based approach to proportions heavily grounds students in multiplicative contexts so that they can build deep knowledge of proportions as multiplicative relationships. Students also experience a broad range of different proportion situations so that they create a robust notion of where proportions might be used. Students work with ratio tables so that they see how proportions come from whole tables of ratios. All initial work is with ratios written horizontally so that ratios are differentiated from fractions.

Many approaches to proportion for fifth graders use unit rates and solve only simple problems that are multiples of the unit rates or of the simple basic ratios. Our approach enables students to solve more complex problems by setting up a Factor Puzzle made from the two rows of a Ratio Table that contain the proportion. At the end the simpler problems are also discussed, but our more general approach facilitates deeper and more general understanding and empowers students to solve more difficult problems.

Begin with Multiplication Column Tables and Situations

Students begin with this problem:

Noreen started to save money. Everyday she put three \$1 coins into her duck bank.

Fill in the Multiplication Column Table to show how much money Noreen saved each day and how much her total was each day.

Multiplication Column Table

Days	Dollars	
0	0	
1	3	+3
2	6	+3
3	9	+3
4	12	+3
5	15	+3
6	18	+3
7	21	+3
8	24	+3

Teaching Unit 6 (Continued)

Students discuss many other situations to decide if they are multiplication column (proportional) situations. They find the unit and the group (the unit rate) for each situation. For example:

Each fish tank has 4 snails to help keep the tanks clean. The unit is 1 tank and the group is 4 snails, so the unit rate is 4 snails per (1) tank.

They also practice the multiplicative language used for rates (*per, for each, every*) and see that a unit rate has a unit of 1 that is not explicitly stated: We say *4 snails per tank* rather than the more specific *4 snails per 1 tank*. Students make Multiplication Column Tables for situations. They also identify whether a given situation and table is a Multiplication Column Table and situation.

Students then discuss the characteristics for the story situations that make a Multiplication Column Table in which the first column is the 1s column and the second column is another column from the multiplication table. These characteristics are that the situation must:

- start with zero
- include a unit that counts the constant increase (for example, 1 row, 1 day, 1 shelf)
- include a group that gives the unit rate (for example, 3 people for each row, 4 snails per fish tank, and 6 flowers per row)
- use the unit rate to make a constant change for each unit (for example, 5 milliliters each 1 day, not 5 one day and 7 the next)

Linked Stories Are Ratios

Students then put together two Multiplication Column stories and tables they have made separately. The columns are linked by their common unit, and each column uses its own rate. In the example on the next page, Noreen's rate is \$3 per day and Tim's rate is \$5 per day. Their common unit is day. After this, students easily see that a Ratio Table is two columns from the multiplication table that are linked by a common unit that is not shown in the table.