Math Background

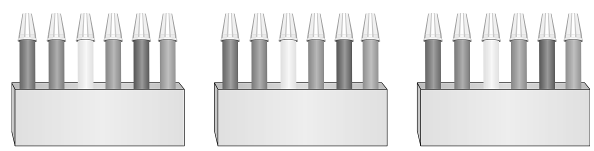
Multiplying by a Fraction

We develop fraction meanings by relating them to the equal groups meaning of multiplication.

Markers come in sets of 6.
Alta has 3 sets.

6 taken 3 times = ____________ markers

$3 \times 6 =$ ____________ markers



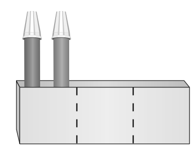
3 sets of 6

Isabel has $\frac{1}{3}$ of a set of 6 markers.

6 taken $\frac{1}{3}$ times = ____________ markers

$\frac{1}{3} \times 6 =$ ____________ markers

$\frac{1}{3}$ of $6 = \frac{1}{3} \times 6 = 6 \div 3 = \frac{6}{3} = 2$



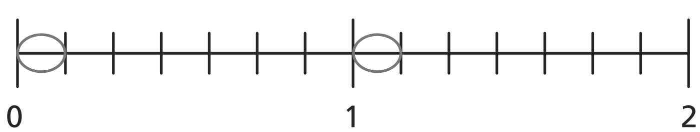
$\frac{1}{3}$ set of 6

Because $\frac{1}{3}$ is 1 of 3 equal parts, finding $\frac{1}{3} \times 6$ requires dividing 6 into 3 equal parts.

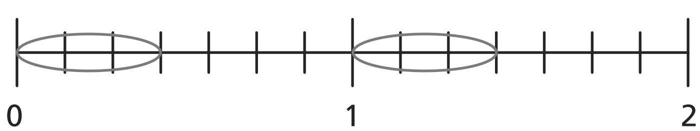
Students review multiplication comparison problems, where the $\frac{1}{3}$ times as many language helps to establish the meaning of multiplying by a unit fraction, $\frac{1}{d}$, as meaning dividing by *d* (the reciprocal of $\frac{1}{d}$).

Students solve more difficult fractional multiplication problems that do not divide evenly, by multiplying *each* 1 whole or *each* unit fraction that makes the multiplied quantity.

$\frac{1}{7} \times 2 = \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$



$\frac{3}{7} \times 2 = \frac{3}{7} + \frac{3}{7} = \frac{6}{7}$



They also see that finding $\frac{n}{d} \times w = n \times \frac{1}{d} \times w = n \times \frac{w}{d} = \frac{n \times w}{d}$.

In the second example above, with $w = 2$: $\frac{3}{7} \times 2 = 3 \times \frac{1}{7} \times 2 = 3 \times \frac{2}{7} = \frac{3 \times 2}{7}$.

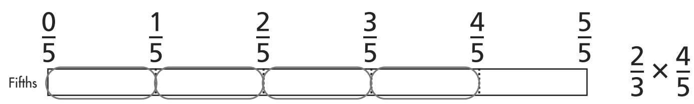
In all of this work, students are asked to observe that multiplying by a fraction makes a smaller number because you are taking only a part of the whole number. This is true for proper fractions.

Multiplying a Fraction Times a Fraction

Students work on their MathBoards solving real-world problems and then find the general pattern for multiplying fractions across repeated examples.

In Lesson 4, for example, you will tell the class that a board is $\frac{4}{5}$ yd long. Ask how long $\frac{2}{3}$ of the board would be. On the front of the MathBoards, students will write $\frac{2}{3} \times \frac{4}{5} =$ to the right of the fraction bar for fifths.

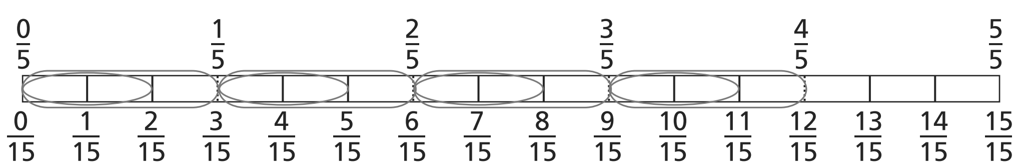
You will begin by labeling the bar and circling four of the fifths.



The class will discuss how to find the denominator:

- How can you show $\frac{2}{3}$ of the circled $\frac{4}{5}$? It is difficult to take $\frac{2}{3}$ as a chunk, but you can take $\frac{2}{3}$ of each fifth.
- If you 3-split each fifth, how many parts will you have? 15
- What denominator will our answer have? 15

Then students will label the new divisions (15ths) below the bar and circle $\frac{2}{3}$ of each of the four circled fifths to find the numerator.



Discussion of what the fraction bar shows will lead to the product.

- How many fifteenths are circled? How do you know? 8; there are 4 groups of 2 circled.
- What is the answer to $\frac{2}{3} \times \frac{4}{5}$? $\frac{2 \times 4}{3 \times 5} = \frac{8}{15}$
- When you take a fraction of a fraction, you are dividing the whole into smaller parts. Why do you get a larger denominator? A larger denominator means that the whole is divided into more but smaller parts.
- When $\frac{a}{b} < 1$ (that is, $a < b$), the product will be less than $\frac{c}{d}$ because you will only take part of ($a \cdot c$ of) the new unit fractions $b \cdot d$.

General Pattern

$\frac{a}{b} \times \frac{c}{d}$

Find the denominator:

b-split each d^{th}

$\frac{a}{b} \times \frac{c}{d} = \frac{}{b \cdot d}$

$b \times d$ is the new denominator.

Find the numerator:

c groups of *a*

$\frac{a}{b} \times \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

$a \times c$ is the new numerator.

Find the product:

$\frac{\text{multiply tops}}{\text{multiply bottoms}}$