Math Background

Shift Patterns for Multiplying by Whole Numbers and by Decimals Less Than One

Students see numbers expressed as money (dollars and coins for different place values) and discuss the effect of multiplying a 3-digit number by 10, 100, and 1,000. They see that each place in the number will get 10 times larger when multiplied by 10 so the whole number will move one place larger (left). Multiplying a number by 100 shifts it 2 places larger (left) and by 1,000 shifts it 3 places larger (left). Multiplying by decimal numbers 0.1, 0.01, and 0.001 results in the opposite shifts: the number gets 1, 2 , and 3 places smaller (it shifts that many places to the right). These shift rules based on thinking about values of quantities are used in multiplying whole numbers and in deriving a pattern for multiplying by a decimal number (add the number of places in the factors to find the number of places in the product).

Methods of Multidigit Multiplication

Students use an area model to see how place values in each factor multiply each other. They develop their own numerical methods and also discuss the following 3 research-based accessible algorithms and the common but complex short-cut method.

Rectangle Sections

43×67	60	$+$	7	
40	$40 \times 60 = 2{,}400$		$40 \times 7 = 280$	40
$+$				$+$
3	$3 \times 60 = 180$		$3 \times 7 = 21$	3
	60	$+$	7	

Expanded Notation

$$\begin{array}{r} 67 = 60 + 7 \\ \times\ 43 = 40 + 3 \\ \hline 40 \times 60 = 2{,}400 \\ 40 \times\ \ 7 = \ \ 280 \\ 3 \times 60 = \ \ 180 \\ 3 \times\ \ 7 = \ \ \ 21 \\ \hline \ \ \ \ \ \ 2{,}881 \end{array}$$

Rectangle Rows

43×67	67	
40	$\overset{2}{67}$ $\times\ 40$ $2{,}680$	$2{,}680$
$+$		$+\ 201$
3	$\overset{2}{67}$ $\times\ 3$ 201	$2{,}881$

Short Cut
Multiply by Tens First

$$\begin{array}{r} \overset{2}{67} \\ \times\ 43 \\ \hline 2{,}680 \\ 201 \\ \hline 2{,}881 \end{array}$$

Short Cut
Multiply by Ones First

$$\begin{array}{r} \overset{2}{67} \\ \times\ 43 \\ \hline 201 \\ 2{,}680 \\ \hline 2{,}881 \end{array}$$

The top two methods show clearly what happens with each partial product, and the bottom two methods are shorter methods; but the Short Cut method is challenging for many students.

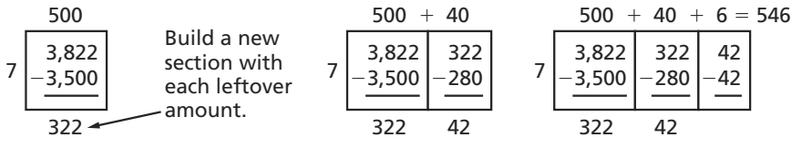
Students solve and generate multiplication problems for all 4 types of multiplication situations. They then extend their methods to 3-digit times 3-digit multiplication. For these problems the top two methods remain possible but have long columns of totals to add. Many students find the Rectangle Rows method to be the best combination of accessibility and efficiency.

Methods of Multidigit Division

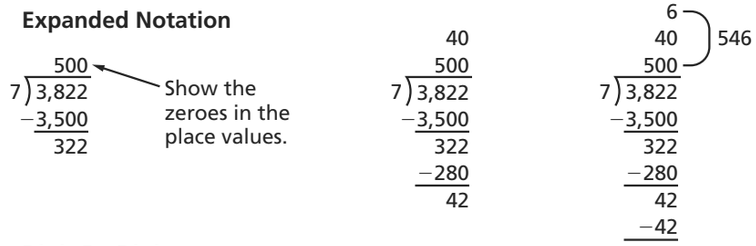
Students develop their own methods of dividing whole numbers and also discuss the following 2 research-based accessible algorithms and the common Digit-By-Digit method.

An airplane travels the same distance every day. It travels 3,822 miles in a week. Compare these methods of dividing that can be used to find how many miles the airplane travels each day.

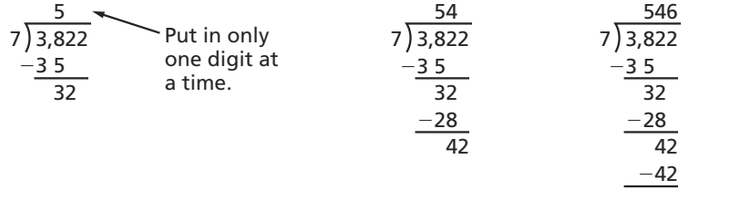
Rectangle Sections



Expanded Notation



Digit-By-Digit



Meanings of Remainders and Dividing by Decimal Numbers

Students explore 5 different meanings a remainder can have in real-world situations. They express fractions as decimals including as repeating decimals. They examine shift patterns for dividing by decimals less than one and see shifts opposite to those made by multiplying by a decimal number. Dividing by 01, 0.01, and 0.001 shifts the number 1, 2, and 3 places larger (it shifts that many places to the left) because you are finding how many small decimal parts are in a number. Students then find general patterns for dividing by a decimal number by multiplying the top and bottom of a division problem by 1 in a form that changes the decimal divisor to a whole number (e.g., for a divisor of 0.47, multiply by 100/100).

Continue to use the multiplication and division fluency materials as needed (see page 345J for a summary).