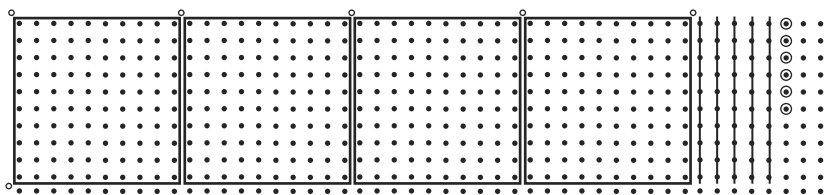


Math Background

Place Value

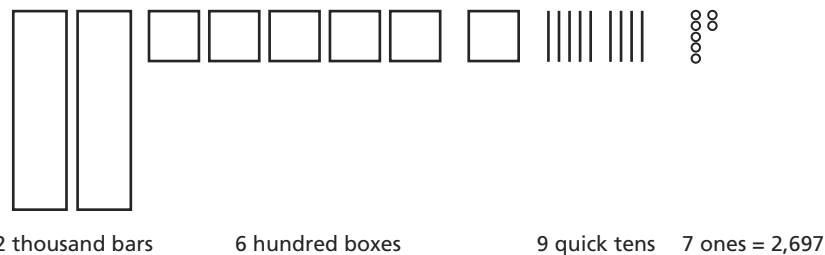
In *Math Expressions*, students use place-value drawings to help them conceptualize numbers and understand the relative sizes of place values. In the early lessons of this unit, students make these drawings on the dot-array side of their MathBoards. The drawing below represents the number 456. It shows

- 4 hundred boxes (2 squares that each contain 100 dots) = 400
- 5 quick tens (5 line segments that each connect 10 dots) = 50
- 6 ones (6 circles that each contain 1 dot) = 6



The place-value drawing helps students understand, for example, that the 4 in the hundreds place actually represents 400.

Once students have a conceptual understanding of the number of 1s contained inside each place, they move to drawings without dots. For example, the drawing below shows 2,697. Because these types of drawings do not need to be perfectly scaled, students can make them quickly. (Grouping the shapes by fives, as in this drawing, makes them easier to count.)

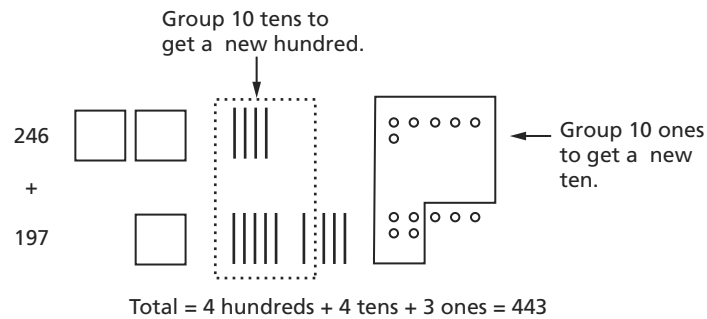


Methods of Addition and Subtraction

Most grade 4 students know how to add and subtract ones and tens, perhaps also hundreds. However, some students go through the motions without understanding why they do what they do. In this unit, students explore the conceptual bases for multi-digit addition and subtraction.

For addition, *Math Expressions* uses the term *grouping* to refer to the process of combining 10 units of one place value to get 1 unit of the next greater place value (for example, combining 10 ones to make a new ten or combining 10 tens to make a new hundred). This process is often referred to as “carrying” in traditional curricula. The opposite process, referred to as *ungrouping*, is used for subtraction. Ungrouping involves breaking apart 1 unit of a place value to get 10 units of the next smaller place value (for example, breaking apart 1 hundred to get 10 tens or breaking apart 1 ten to get 10 ones). This process is often referred to as “borrowing” in traditional curricula.

Drawings for Addition Students use place-value drawings to help them understand the process of grouping. To model $197 + 246$, make place-value drawings for the two addends. Then, group 10 ones to make a new ten and 10 tens to make a new hundred. After grouping, the drawing shows 4 hundreds, 4 tens, and 3 ones, or 443.



Numeric Addition Methods Students connect the place-value drawings with numeric methods. The examples below show three numeric methods for adding 246 and 197.

New Groups Above

$$\begin{array}{r} 11 \\ 246 \\ + 197 \\ \hline 443 \end{array}$$

New Groups Below

$$\begin{array}{r} 246 \\ + 197 \\ \hline 443 \end{array}$$

Show Subtotals

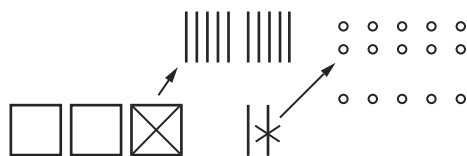
$$\begin{array}{r} 246 \\ + 197 \\ \hline 300 \\ 130 \\ \hline 13 \\ \hline 443 \end{array}$$

The New Groups Above method is the method commonly taught in U.S. schools. The New Groups Below method is identical to New Groups Above, except the 1s indicating the new groups are written under the addends. In the Show Subtotals method, the hundreds, tens, and ones are added separately and then the three subtotals are added to get the final sum.

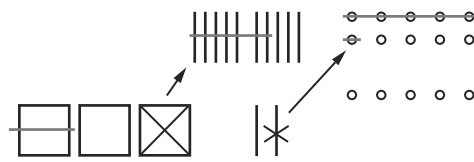
Drawings for Subtraction To make place-value drawings for subtraction, draw the number you are subtracting from and take away the number you are subtracting. For example, to find $325 - 176$, start with the drawing for 325.



You must take away 1 hundred, 7 tens, and 6 ones. There are not enough tens or ones in the drawing to do this, so you must ungroup a ten to get more ones and ungroup a hundred to get more tens.



After crossing-out 1 hundred, 7 tens, and 6 ones, you are left with 1 hundred, 4 tens, and 9 ones, or 149.



$$1 \text{ hundred} + 4 \text{ tens} + 9 \text{ ones} = 149$$

Numeric Subtraction Method Students are encouraged to do all the necessary ungrouping before subtracting. Students can ungroup either from left to right or from right to left. Once everything is ungrouped, students can subtract the places in any order.

Ungroup from Left to Right

$$\begin{array}{r} 325 \\ -176 \\ \hline 149 \end{array}$$

A magnifying glass is positioned over the 325, showing the digits 3, 2, and 5 with a circled 11 above the 2 and a circled 15 above the 5, indicating the ungrouping process.

Ungroup from Right to Left

$$\begin{array}{r} 325 \\ -176 \\ \hline 149 \end{array}$$

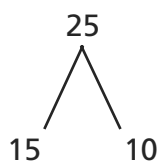
A magnifying glass is positioned over the 325, showing the digits 3, 2, and 5 with a circled 11 above the 2 and a circled 15 above the 5, indicating the ungrouping process.

We encourage you to allow students to use any addition and subtraction methods that work for them, as long as they can explain why the methods work. However, research has shown that the New Groups Below and Show Subtotals methods help students who are having trouble with addition, and ungrouping everything first and then subtracting helps students avoid common subtraction mistakes.

Connecting Addition and Subtraction

Beginning in grade 1, *Math Expressions* builds understanding of the inverse nature of addition and subtraction. A conceptual support for this idea is a break-apart drawing (referred to as a Math Mountain in earlier grades). A break-apart drawing shows a total at the top and two addends at the bottom. Students learn that they can add the two addends to get the total or subtract either addend from the total to get the other addend. They learn that there are eight equations associated with any break-apart drawing.

Break-Apart Drawing



Related Equations

$15 + 10 = 25$	$25 = 15 + 10$
$10 + 15 = 25$	$25 = 10 + 15$
$25 - 15 = 10$	$10 = 25 - 15$
$10 - 10 = 15$	$15 = 25 - 10$

Problem Solving

Throughout the unit, real-world situations are used as the context for problems. Students apply strategies for addition and subtraction strategies to solve various word problems, including problems of change, collection, and comparison.