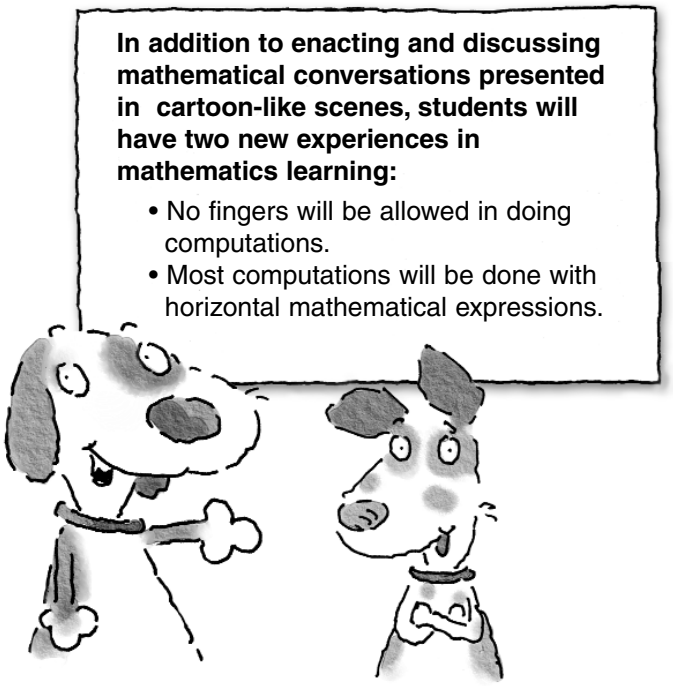
Knowing Mathematics
Program Philosophy

Experience learning in a new way, build on old knowledge
Knowing Mathematics is a small-group mathematics intervention program for fourth- through sixth-graders who are two or more years below grade level. Combining best practices of East Asian and U.S. instruction, it is designed to vertically accelerate students to grade level.
Students are provided with a new way of learning mathematics, different from their previous experiences that may have been accompanied by failure and frustration. At the same time, the curriculum draws on the mathematical knowledge that students already have, although fragmentary and insufficient, to repair and re-organize it to build a sound foundation for future learning.

Our Approach: Mathematical Conversations
The core part of each lesson is delivered through mathematical conversations shown on the first page of each lesson. The conversations, presented by the program's cast of characters and acted out by students, model a way of thinking and learning mathematics that may be new to low-achieving students. They also offer a unique way to get students to talk about mathematical ideas and concepts.



Ms. Park's students express their ideas about mathematics. Some of these ideas are not entirely correct or not correctly expressed. These mistakes and misconceptions are discussed in the conversation or in the lesson support. We realize that this may be considered unusual. However, research from cross-national studies suggests that classroom discussion of incorrect ideas is valuable for student learning.

Studies of elementary classrooms in East Asian countries and the United States reveal different classroom practices. Consistent with the practices described in The Learning Gap,[1] James Stigler, Clea Fernandez, and Makoto Yoshida note that in contrast with Japanese teachers, "American teachers go to great lengths to keep errors out of the mathematics classroom, and especially out of the public discourse." In contrast, Japanese teachers "see errors as a natural part of the learning process and as important sources of information about children's mathematical thinking."[2] It may be for cultural reasons such as fear of damaging students' self-esteem that U.S. teachers do not discuss students' errors. Ms. Park's students reveal common understandings and errors in order that the real students reading the conversations can discuss them and understand why the errors are errors.

Our Approach: Organization of the Mathematics
The lessons are designed to reveal mathematical structures that students may have missed in their earlier experiences, and to gradually build students' understanding of concepts and their skill in using these concepts. By "gradually," we don't mean that we introduce a concept in its final form and expect students to catch on gradually, but that we introduce "seeds" of concepts, and develop and expand on those seeds over the course of several lessons or even several units.[3]

For example, Lesson 1 of Unit 1: Addition and Subtraction, begins with a discussion of the idea of a number as the amount of objects in a set. That leads to the idea of composing and decomposing numbers, joining several sets into a larger set or breaking one set into several smaller sets. The idea of composing and decomposing numbers is used throughout the units on numbers (and composing and decomposing a shape is used in the geometry units). Lesson 2 introduces the idea of 10 as an organizer of the number system and begins work with composing and decomposing a ten. The skill of composing and decomposing a ten and the idea of 10 as an organizer are used when students learn to add and subtract with "carrying and borrowing." The idea of 10 as an organizer is used during the multiplication and division unit (Unit 2), and further refined in the place value unit (Unit 3).

Our Approach : Mathematical Language
The language used in this book is designed to help reveal mathematical structures to students and to be consistent with later mathematical usage.

For example, to help reveal mathematical structures we use **"composing and decomposing"** instead of "carrying and borrowing," or "regrouping." Their names suggest that each is the inverse of the other: composing is the inverse of decomposing and decomposing is the inverse of composing. Moreover, "composing and decomposing" can indicate what is composed and decomposed—a ten can be decomposed into 10 ones, a hundred can be decomposed into 10 tens, or 11 can be decomposed as 10 ones and 1 one.

We use **"expressions"** and **"equations"** rather than "number sentences." These are consistent with later usage. We talk about "using an expression to represent a mathematics problem" or "using a model to represent a problem." In some instances, we use "compute" and "calculate" rather than "add," "subtract," "multiply," and "divide," distinguishing between doing an operation and evaluating the result of the operation. For instance, we say, "calculate 2 + 4," rather than "add 2 + 4." [4]

Lesson Format
Each two-page lesson of **Knowing Mathematics** is written for one instructional session (40 to 50 minutes in length) and contains the following parts:

- **Warm-Up Exercises:** These exercises prepare students for the topic of the lesson or act as a review of previous lessons.
- **Getting Ready for the Mathematical Conversation:** After the Warm-Up exercises, students briefly discuss the lesson title. The title is designed to focus on the main topic of the lesson and is often a question so students are encouraged to predict what they will learn.
- **Mathematical Conversation:** The main part of the first page, the instruction page, shows a mathematical conversation between the teacher (Ms. Park) and her students. Students read and act out the conversation, stopping for discussions at strategically placed locations. The instruction page is designed for students to think about the mathematics they are learning.
- **Guided Practice:** As a group, with teacher guidance, students ask questions as they try and discuss exercises based on the topic of the lesson. Teaching support for these exercises is scaffolded.
- **Independent Exercises:** For the second part of the lesson, students work independently on exercises that are monitored by the teacher. Acting as a coach, the teacher asks students to discuss their strategies or to notice aspects of the exercises at strategically placed pauses. The exercises are carefully designed to support the learning of the mathematical topic just discussed. Review sections are always included to maintain coherence in learning. The teaching support fades for some topics (such as word problems) over the course of several lessons and for some topics over a single lesson.
- **Reflect and Discuss:** Students share and discuss strategies and opinions about the lesson topic:
  - **Reflection and Discussion** : A problem or question is posed to students that is designed to provide insight into students' understandings of the lesson topic.
  - **Return to the Lesson Title**: Students return to the lesson title where they are encouraged to summarize what the lesson was about and monitor their progress.

You will notice that throughout the instructional model, students are asked for their strategies and opinions. This approach is supported by research in psychology by Ann Brown and colleagues paraphrased here:[5]

- Verbal reports—even without feedback from another person—can benefit learning and problem solving. Most commonly this occurs when the type of report required is a reason for an action or a statement of a rule—possibly a rule created by the learner.
- Good problem solvers identify and evaluate their problem-solving efforts more than poor learners, stating rules and evaluating their efficiency.
- Instructions to state a rule accelerate learning.
- Instructions to state a rule facilitate transfer—students are more likely to use their knowledge in different contexts.
- Learning an explanation for a procedure and the procedure makes transfer more likely.

Aspects of the instructional format are similar to that of reciprocal teaching (used in the reading interventions Early Success and Soar to Success):

(1) Students work in a small group together with a teacher.
(2) The whole group focuses on reading and understanding the instruction page.
(3) Instead of the routines of predict, clarify, question, and summarize used in reciprocal teaching, some features of mathematics learning are made visible by the cast of characters: asking questions about the content, predicting future topics, and connecting past learning with current topics.

Teaching Guide Format

In the margins of the instruction pages are detailed step-by-step suggestions for how to teach the lesson: where to stop, what questions to ask, what answers students might give, and possible replies. In the margins of the exercise pages, detailed instructions are given for guiding and pacing students as they do the exercises. The first lesson in Units 1, 2, and 3 contain a complete stepped-out instructional plan intended to help you learn how to use the program as you are in fact teaching it.

How to Use *Knowing Mathematics*

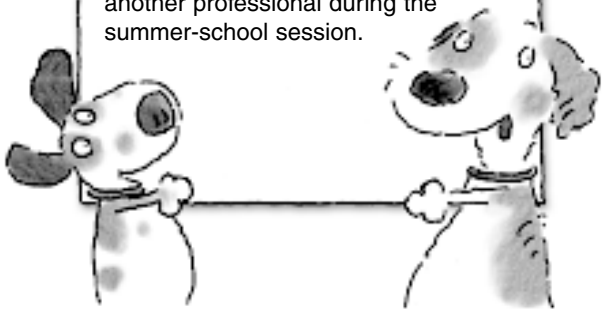
We suggest three different ways in which to use *Knowing Mathematics*. No matter what model is used, lessons need 40 to 50 minutes each day. For most lessons, we suggest that you spend about two fifths of the time on the instruction page and three fifths on the exercise page. To fit the school day well, you may split daily instruction into two segments.

Remember that this is a small-group project. For the pullout model and extended-day model, we suggest that you have not more than eight students, while four to six will be an ideal size. For the summer-school model, we suggest that you have no more than twelve students.

Pullout Model
Students go to another teacher for Knowing Mathematics instruction.

Extended-day Model
Knowing Mathematics instruction is given before or after school by a classroom teacher or another professional.

Summer-session Model
Knowing Mathematics instruction is given by a classroom teacher or another professional during the summer-school session.



Bibliography

[1] Stevenson, Harold & James Stigler, (1992). *The Learning Gap: Why Our Schools Are Failing and What We Can Learn From Japanese and Chinese Education.* New York: Simon and Schuster.

[2] Stigler, Fernandez, & Yoshida, (1996), "Cultures of mathematics instruction in Japanese and American elementary classrooms," in Thomas P. Rohlen & Gerald K. LeTendre (Eds.), *Teaching and Learning in Japan* (pp. 213–247). Cambridge: Cambridge University Press. See also, Liping Ma, *Knowing and Teaching Elementary Mathematics,* 1999, Mahwah, NJ: Lawrence Erlbaum Associates, pp. 48–51.

[3] For more discussion of this idea see Liping Ma, Knowing and *Teaching Elementary Mathematics*, 1999, Mahwah, NJ: Lawrence Erlbaum Associates, pp. 116–119.

[4] For more discussion, see Liping Ma, *Arithmetic in American mathematics education: An abandoned arena?* Plenary Address at National Summit on the Mathematical Education of Teachers, 2001. Text available at: http://www.cbmsweb.org/NationalSummit/Plenary_Speakers/ma.htm.

[5] See page 110 of Ann Brown, John Bransford, Roberta Ferrara, & Joseph Campione, "Learning, Remembering, and Understanding," 1983, in *Handbook of Child Psychology: Vol. 3. Child Development* (4th ed., pp. 77–166), NY: Wiley.

[6] See the discussion of transfer in either edition of *How People Learn: Brain, Mind, Experience, and School,* Washington, DC: National Academy Press.